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METHOD FOR EXPANDING THE OBJECT BASE OF EXAMINATION BY STITCHING SOLUTIONS OF HIERARCHY ANALYSIS METHOD

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The paper presents the results on numerical modelling of the quality of opto-electronic detectors. In order to demonstrate a successful application of the proposed method of the object base extension, we use examples of hierarchy analysis of generalized quality index and integrated quality-price index. The proposed methodology allows reliable analysis the number of objects up to 21–24, that is enough for the most practical cases of examination.

Keywords: hierarchy analysis method; optoelectronic detectors; quality index.

Introduction

In his fundamental work [1] T. Saati conducted detailed statistical and psychological research of validity and reliability of the hierarchy analysis method (Principal Component Analysis – PCA), based on the theory of fuzzy sets. Justifying his theory, T. Saati wrote: the process "...is usually called *hierarchy*, i.e., a system of layered layers, each of which consists of many elements, or factors. The central question in the language of hierarchy is the following: how much individual factors having the lowest level of the hierarchy influence on the top (that is, on the common goal). The influence of all factors is unequal, therefore it is necessary to determine an intensity of the influence, or, as we prefer to say, *priorities of factors*. The determination of the lower-level factors priorities as to the goal can be reduced to a sequence of problems about determination of priority for each level, and each such problem can be reduced to a sequence of pairwise comparisons. ...The theory a model of the natural course of human thinking ... " [1, p. 5].

However, in spite of the thoroughness of mathematical, statistical and psychological study of the PCA method, the method has a serious drawback, which pointed out by the author himself. So, in his work [1], T. Saati gives the values of 7 ± 2 as the maximum number of compared objects, and there are only 9 ranks in the linguistic scale (1, 2, ..., 9). Thus, an *object base* (that is, a set of compared objects with their technical characteristics) is very limited.

Such a restriction, according to T. Saati, is conditioned by the psychological characteristics of a person, and not by the mathematical difficulties. It is proved that a person can not effectively assess the differences of more than 7-8 objects, and the proposed nine-point scale of ranks was based on a double application of the trichotomy principle.

Also, let us note another problem of using the method. Let W be a matrix of pairwise comparisons, which is constructed at the first step of the work. The problem is a coordination of the matrix W. In order to estimate the coordination of W, the coordination index $CI = (\lambda_{\max} - m) / (m - 1)$ and the coordination relation $CR = CI/CV$ (where CI is a coordination index, CR is a coordination relation, CV is a coordination value for a random matrix having the same order, and both values (CI and CR) should be not more than 0,1) are calculated.

The average values of the coordination CV for random matrices having different orders, obtained by randomly choosing quantitative paired estimates of relative importance from the scale 1/9, 1/8, 1/7, ..., 1, 2, ..., 9 and the formation of the inverse symmetric matrix W, are given in [1, 3] (Table 1).

Table 1

Values of the coordination for random matrices having different orders

Matrix order	3	4	5	6	7	8	9	10
Random coordination	0,58	0,90	1,12	1,24	1,32	1,41	1,45	1,49

It is very difficult to achieve acceptable coordination in matrices of large sizes, as from 7–9 elements. In addition, the values of random error given in Table 1 for the number of compared objects $7 \div 10$, are simply unacceptable.

1. Modifying the PCA Method to Extend the Object Base

In order to ensure the applicability of the PCA for a number of objects larger than 7–8, the method of expanding the object base of examination by joining the hierarchy analysis solutions is proposed. The method is the following.

Suppose it is necessary to compare L objects of examination (factors)[1] $Z = (z_1, z_2, ..., z_L)$, where $L \geq 10$. Then the following algorithm is proposed.

1. Choose the maximum of the set Z elements. In future, $z_{\max}$ is used as a reference element.

2. Divide the set Z into subsets Z_i such that for any i the number of elements in Z_i is not more than 5–6.

3. For each subset Z_i, construct a matrix of pairwise comparisons W_i by usual PCA method, in which the same $z_{\max}$ is used as a reference element.

4. Find eigenvalues and eigenvectors for all constructed matrices of pairwise comparisons W_i.

5. Compare the maximum values of the obtained priority vectors with each other and determine the coefficients of difference k_i.

6. Multiply all elements of the i-th priorities vector on the coefficient of difference.

7. Draw a graph of the total priority vector for the entire set Z.

Thus, the proposed method allows to increase the number of compared objects up to 18–20. A simplification of the coordination of pairwise comparisons matrices W_i is one of the proposed method merits. In addition, according to Table 1, in the case of matrices having lower dimension (6×6) the random error decreases from 0,49 to 0,24 in comparison with a matrix (10×10).

[1] In the paper, the factors are variables x_j, weighted sums of individual feature groups, quality indexes J, J_k, etc.

2. An Hierarchy of Generalized Quality Indexes J_q

In order to illustrate the proposed algorithm, we first consider a set of generalized quality indexes J_q of opto-electronic detectors calculated in [5].

Let us illustrate the application of the proposed method of joining the results of the hierarchy analysis method (PCA). As an example, we consider comparing the characteristics of industrial fire alarm detectors. According to the examination procedure developed in [4], the following features of examination objects are pointed out: *quantitative features* (horizontal viewing angle, range, notification time), *existence features* (anti-sabotage zone existence, possibility to adjust the sensitivity), *qualitative features* (type of detection zone).

After normalizing the characteristics and determining the weight coefficients of the quality functional, the following normalized values were calculated in [5]: the generalized quality index J_q, the cost index P and the complex index "quality-price" J for 10 compared devices (Table 2).

Table 2

Indexes of 10 compared devices

Detectors	IO309-11 "Astra-5" performed by B1	IO209-24 "Astra-5" performed by B2	IO209-20 "Photon-10A"	IO209-21 "Photon-15A"	IO209-27 "Photon-16A"
Device numbers	1	2	3	4	5
Generalized quality index	0,592	0,629	0,335	0,464	0,581
Cost index	1,000	1,000	0,747	0,585	0,558
Complex index	0,796	0,814	0,541	0,525	0,570

Detectors	IO 309-28 "Astra-531" performed by IK	IO309-19 "Ikar-Sh"	IO309-16 "Ikar-5B"	IO309-9 "Photon-10B"	IO309-10 "Photon-15B"
Device numbers	6	7	8	9	10
Generalized quality index	0,738	0,547	0,586	0,428	0,299
Cost index	0,935	0,745	0,494	0,747	0,584
Complex index	0,836	0,646	0,541	0,588	0,442

First, we draw a graph of J_q for 10 samples according to the data in Table 1 (see Fig. 1). According to the proposed method, we divide the set of detectors into two subsets, which according to the data in Table 2 for generalized quality indexes J_q (factors) take the form

$$Z_1 = \begin{pmatrix} 0,592 & 0,629 & 0,335 & 0,464 & 0,581 & 0,738 \end{pmatrix}^T, \tag{1}$$

$$Z_2 = \begin{pmatrix} 0,547 & 0,586 & 0,428 & 0,299 & 0,738 \end{pmatrix}^T. \tag{2}$$

Note that both groups include the same reference element $z_{\max} = 0,738$.

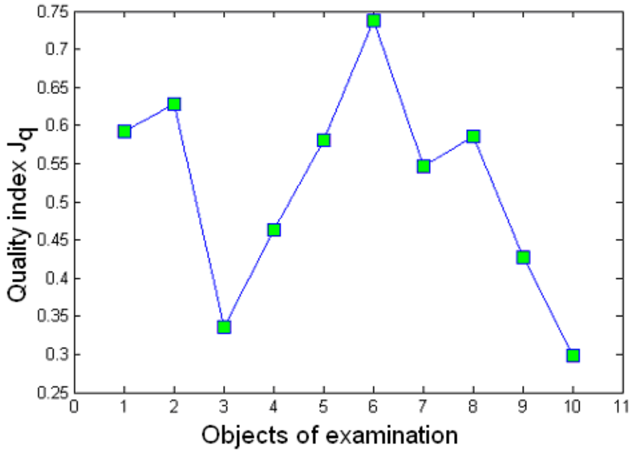

Fig. 1. Generalized quality index J_q

Let us construct the matrices of pairwise comparisons for both groups. According to the usual PCA method, elements of the vectors (1), (2)are preliminary preordered in descending order, matrix of pairwise comparisons is determined, eigenvalues and eigenvectors are found, and then a reverse transition to the original order of factors is performed.

For the ordered elements of the first group, we obtain

$$W_1 = \begin{pmatrix} 1 & 3 & 4 & 4 & 5 & 6 \\ 0,33 & 1 & 2 & 2 & 4 & 5 \\ 0,25 & 0,5 & 1 & 1 & 3 & 4 \\ 0,25 & 0,5 & 1 & 1 & 2 & 4 \\ 0,2 & 0,25 & 0,33 & 0,5 & 1 & 3 \\ 0,17 & 0,2 & 0,25 & 0,25 & 0,33 & 1 \end{pmatrix}. \tag{3}$$

The maximum eigenvalue for the matrix (3) is 6,233. Coordination index (CI) is 0,047, and coordination relation (CR) is 0,038. Therefore, the matrix W_1 is well coordination.

The first eigenvector of the matrix W_1 (priority vector) has the form

$$V^{(1)} = \begin{pmatrix} 0,827 & 0,414 & 0,257 & 0,236 & 0,133 & 0,075 \end{pmatrix}^T. \tag{4}$$

Let us construct the matrix of paired comparisons for the second group of factors ordered by decreasing:

$$W_2 = \begin{pmatrix} 1 & 4 & 4 & 5 & 6 \\ 0,25 & 1 & 1 & 3 & 5 \\ 0,25 & 1 & 1 & 3 & 5 \\ 0,20 & 0,33 & 0,33 & 1 & 3 \\ 0,17 & 0,20 & 0,20 & 0,33 & 1 \end{pmatrix}. \tag{5}$$

The maximum eigenvalue for the matrix (5) is 5,249. Coordination index (CI) is 0,062, and coordination relation (CR) is 0,055. Therefore, the matrix W_2 is well coordinated.

The first eigenvector of the matrix W_2 (priority vector) has the form

$$V^{(2)} = \begin{pmatrix} 0,878 & 0,321 & 0,319 & 0,145 & 0,076 \end{pmatrix}^T. \tag{6}$$

It is easy to see that the maximum elements in the expressions (5) and (6) are different. In order to join the MAI decisions, we define the coefficient of difference for the second group (that is, $k_2 = 0,942$) and multiply the vector $V^{(2)}$ by k_2. We obtain

$$V^{(2)} = \begin{pmatrix} 0,827 & 0,303 & 0,300 & 0,136 & 0,072 \end{pmatrix}. \quad (7)$$

Recall that the vectors $V^{(1)}, V^{(2)}$ were obtained by the PCA method after the ordering of the factors by decreasing. We reverse the numbering of the factors in accordance with their order in the initial set of detectors, "join" the priority vectors of both groups and finally obtain the vector of the generalized quality index J_q:

$$J_q = \begin{pmatrix} 0,257 & 0,414 & 0,075 & 0,133 & 0,236 & 0,827 & 0,303 & 0,300 & 0,136 & 0,072 \end{pmatrix}^T. \quad (8)$$

Graphically represent the quality index J_q obtained by the modified PCA method (Fig. 2). It is easy to see that the difference between "good" and "bad" detectors is more contrasting. It seems to be an additional convenience for the person who takes managerial decisions when examining.

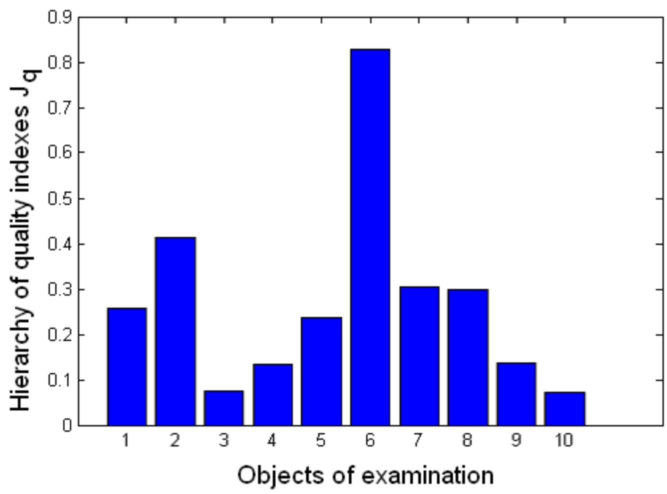

Fig. 2. Generalized quality index for modified method of hierarchies analysis

3. The Hierarchy of Complex Quality-Price Indexes J

Now we consider a set of complex quality-price indexes J for 10 opto-electronic detectors.

First, we draw a graph J for 10 samples according to the data in Table 2 (Fig. 3).

According to the proposed method, we divide the set of detectors into two subsets, which according to the Table 2 for the complex quality-price indexes (factors) J have the form

$$Z_1 = \begin{pmatrix} 0,796 & 0,814 & 0,541 & 0,525 & 0,570 & 0,836 \end{pmatrix}^T, \quad (9)$$

$$Z_2 = \begin{pmatrix} 0,646 & 0,540 & 0,588 & 0,442 & 0,836 \end{pmatrix}^T, \quad (10)$$

Note that both groups include the same reference element $z_{\max} = 0,836$.

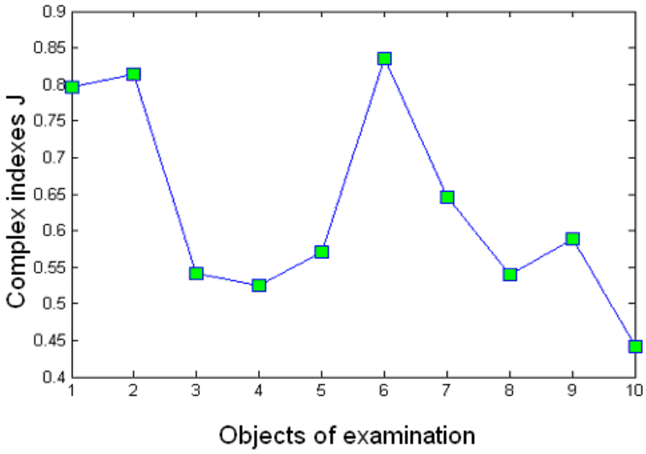

Fig. 3. Complex quality-price index J

Let us construct the matrices of pairwise comparisons for both groups. According to the usual PCA method, elements of the vectors (9), (10) are preliminary preordered in descending order, matrix of pairwise comparisons is determined, eigenvalues and eigenvectors are found, and then a reverse transition to the original order of factors is performed.

For the ordered elements of the first group, we obtain

$$W_1 = \begin{pmatrix} 1 & 1 & 1 & 4 & 4 & 5 \\ 1 & 1 & 1 & 4 & 4 & 5 \\ 1 & 1 & 1 & 2 & 2 & 3 \\ 0,25 & 0,25 & 0,5 & 1 & 1 & 2 \\ 0,25 & 0,25 & 0,5 & 1 & 1 & 2 \\ 0,20 & 0,20 & 0,33 & 0,50 & 0,50 & 1 \end{pmatrix}. \tag{11}$$

The maximum eigenvalue for the matrix (11) is 6,096. Coordination index (CI) is 0,019, and coordination relation (CR) is 0,0038. Therefore, the matrix W_1 is well coordination.

The first eigenvector of the matrix W_1 (priority vector) has the form

$$V^{(1)} = \begin{pmatrix} 0,601 & 0,601 & 0,445 & 0,182 & 0,182 & 0,112 \end{pmatrix}^T. \tag{12}$$

Let us construct the matrix of paired comparisons for the second group of factors ordered by decreasing:

$$W_2 = \begin{pmatrix} 1 & 3 & 4 & 4 & 6 \\ 0,33 & 1 & 2 & 2 & 5 \\ 0,25 & 0,50 & 1 & 1 & 4 \\ 0,25 & 0,50 & 1 & 1 & 3 \\ 0,17 & 0,20 & 0,25 & 0,33 & 1 \end{pmatrix}. \tag{13}$$

The maximum eigenvalue for the matrix (13) is 5,124. Coordination index (CI) is 0,031, and coordination relation (CR) is 0,028. Therefore, the matrix W_2 is well coordinated.

The first eigenvector of the matrix W_2 (priority vector) has the form

$$V^{(2)} = \begin{pmatrix} 0,856 & 0,395 & 0,237 & 0,220 & 0,086 \end{pmatrix}^T. \quad (14)$$

It is easy to see that the maximum elements in the expressions (12) and (14) are different. In order to join the PCA decisions, we define the coefficient of difference for the first group (that is, $k_1 = 1,424$) and multiply the vector $V^{(1)}$ by k_1. We obtain

$$V^{(1)} = \begin{pmatrix} 0,856 & 0,856 & 0,634 & 0,261 & 0,260 & 0,159 \end{pmatrix}^T. \quad (15)$$

Recall that the vectors $V^{(1)}, V^{(2)}$ were obtained by the PCA method after the ordering of the factors by decreasing. We reverse the numbering of the factors in accordance with their order in the initial set of detectors, "join" the priority vectors of both groups and finally obtain the vector of complex quality-price index J:

$$J = \begin{pmatrix} 0,634 & 0,856 & 0,159 & 0,260 & 0,261 & 0,856 & 0,395 & 0,220 & 0,237 & 0,086 \end{pmatrix}^T. \quad (16)$$

Graphically represent the complex quality-price index J obtained by the modified PCA method (Fig. 4).

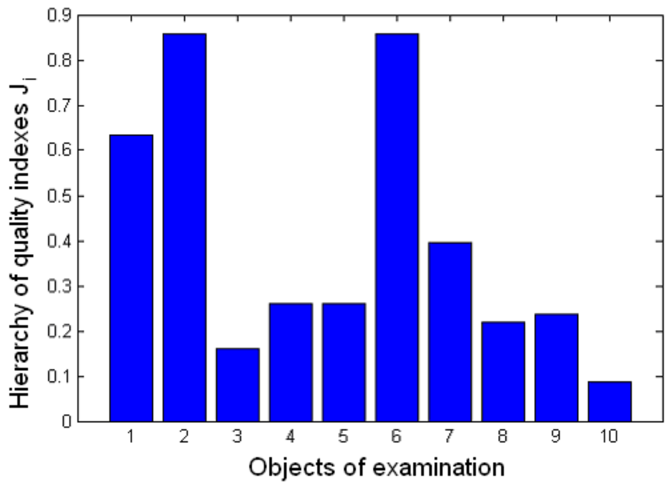

Fig. 4. Complex quality-price index for the modified hierarchy analysis method

Either as in the case of the hierarchies analysis of generalized quality index (see Fig. 2), the difference between "good" and "bad" detectors is more contrasting compared to the set of initial indexes (see Table 2). It seems to be an additional convenience for the person who takes managerial decisions when examining.

Thus, we demonstrated the successful application of the proposed method of extending the object base. As an examples, we considered the hierarchies analysis of the generalized quality index J_q and the complex quality-price index J. It is well known that T. Saati recommended to use the method only in cases when the number of objects is not more than 7–9 [1]. We claim with confidence that the proposed methodology allows reliably analyze the number of objects up to 21–24, that is enough for the most practical cases of examination.

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МЕТОД РАСШИРЕНИЯ ОБЪЕКТОВОЙ БАЗЫ ЭКСПЕРТИЗЫ СШИВАНИЕМ РЕШЕНИЙ МЕТОДА АНАЛИЗА ИЕРАРХИЙ

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В работе представлены результаты численного моделирования качества оптико-электронных извещателей. Продемонстрирована успешность применения предложенного метода расширения объектовой базы на примерах анализа иерархий обобщенного показателя качества и комплексного показателя качество-цена. Предложенная методика позволит проводить достоверный анализ до количества объектов 21–24, что покрывает потребности большинства практических случаев экспертизы.

Ключевые слова: метод анализа иерархий; оптико-электронные извещатели; показатель качества.

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