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COMPUTER SIMULATION OF THE VOLGA RIVER HYDROLOGICAL REGIME: PROBLEM OF WATER-RETAINING DAM OPTIMAL LOCATION

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We investigate of a special dam optimal location at the Volga river in the area of the Akhtuba left sleeve beginning (7 km to the south of the Volga Hydroelectric Power Station dam). We claim that a new water-retaining dam can resolve the key problem of the Volga-Akhtuba floodplain related to insufficient water amount during spring floodings due to the overregulation of the Lower Volga. Using a numerical integration of Saint-Vanant equations we study the water dynamics across the northern part of the Volga-Akhtuba floodplain taking into account its actual topography. As the result we found an amount of water V_A passing to the Akhtuba during spring period for a given water flow through the Volga Hydroelectric Power Station (so-called hydrograph which characterises the water flow per unit of time). By varying the location of the water-retaining dam x_d, y_d we obtained various values of $V_A(x_d, y_d)$ as well as various flow spatial structure on the territory during the flood period. Gradient descent method provides the dam coordinated with the maximum value of V_A. Such approach to the dam location choice let us find the best solution, that the value V_A increases by a factor of 2. Our analysis demonstrates a good potential of the numerical simulations in the field of hydraulic works.

Keywords: hydrodynamic simulation; Saint-Venant equations; numerical model; optimization; hydrology.

Introduction. The unique landscape of 20000 km^2 Volga-Akhtuba floodplain (VAF) depends on special features of the interfluve hydrological regime. During the spring flood period the area between the Volga and Akhtuba rivers is heavily flooded [1–3], that ensures a special composition of flora and fauna and possibility of agricultural use of the area including the development of magnificent gardens and melon fields. The floodplain is also the basis for the fish reproduction at the Lower Volga region [4]. Nowadays the overregulation of the Volga-Kama basin hydrological regime by 22 Hydroelectric Power Plants leads to the VAP degradation.

Various approaches to the problem solution have beed proposed. Let us point out the attempts to construct so-called optimal hydrograph $Q(t)$ [5–7] which is close to the natural and ensures preservation of the natural rate. Despite the progress in the construction of the mathematical and hydrological regime control problems and the territory as a whole [8–11], there is a great difficulty of their practical implementation due to the conflict of various agents aspiration (energetics, environmental protection organizations, fish industry, agriculture, inhabitants, safety of reservoirs, etc.). Over the last decades the situation becomes even more complicated due to changes in the Volga riverbed below the dam mainly because of the violation of the spring flood natural process. In this study we discuss the possibility of the floodplain hydrological regime improvement by the construction of the water-retaining dam at the Volga riverbed close to the beginning of the Akhtuba left sleeve which is located approximately 7 km below the Volga Hydroelectric Power Station dam (Fig. 1). The main aim of the work is construction of a mathematical model for the

Fig. 1. The map of the northern part of the Volga-Akhtuba floodplain with the topography. The insert shows the part of the Volga with the Akhtuba springhead

estimation of the dam optimal location which provides the larger water flow rate in the Akhtuba during the spring flood.

1. The Hydrodynamic Model. At first, we briefly describe the hydrodynamic model underlying the base of our research. We use the Saint-Venant equations for the shallow water dynamics at a given topography $b(x, y)$ [1, 9, 12]:

$$\frac{\partial H}{\partial t} + \frac{\partial uH}{\partial x} + \frac{\partial vH}{\partial y} = \sigma(x, y, t)\,, \tag{1}$$

$$\frac{\partial (uH)}{\partial t} + \frac{\partial (u^2 H)}{\partial x} + \frac{\partial (uvH)}{\partial y} = -gH\frac{\partial (H+b)}{\partial x} + 2vH\Omega_E \sin(\theta) + f_x^{\mathrm{fric}} + f_{\sigma x}\,, \tag{2}$$

$$\frac{\partial (vH)}{\partial t} + \frac{\partial (uvH)}{\partial x} + \frac{(v^2 H)}{\partial y} = -gH\frac{\partial (H+b)}{\partial y} - 2uH\Omega_E \sin(\theta) + f_y^{\mathrm{fric}} + f_{\sigma y}\,, \tag{3}$$

where H is the water depth, u and v are the velocity of x- and y-components (averaged vertically), σ is the source function, g is referred to the gravitational acceleration, Ω_E is the Earth's angular velocity, θ is the latitude, the values of $f_{\sigma x}$ and $f_{\sigma y}$ describe the water impulse associated with the sources σ. For the bottom friction force vector we use the Chezy's model [1]:

$$f_x^{\mathrm{fric}} = -\frac{u}{2}\sqrt{u^2+v^2}\,H\Lambda\,, \quad f_y^{\mathrm{fric}} = -\frac{v}{2}\sqrt{u^2+v^2}\,H\Lambda, \tag{4}$$

with the value of hydraulic friction $\Lambda = 2gn_M^2/H^{4/3}$ and the Manning roughness coefficient n_M.

Value of n_M is determined by the surface properties and generally it depends on the coordinates. Moreover, for the unsteady flow regime the Manning coefficient can vary with time [1]. The hydrograph through the Volga Hydroelectric Power Station dam allows to set the value of $\sigma = \frac{dQ_0}{dS}$ (where $dS = dx\,dy$ is an elementary area). Under the VAP conditions the value of n_M for the Volga riverbed varies in the range 0,02 – 0,07 [1].

For the numerical solution of equations (1) – (3) we apply our combined Lagrangian – Eulerian method (cSPH-TVD) [13] which uses the benefits of Smoothed Particle

Hydrodynamics at different time steps and Total Variation Diminishing (see the detailed description in [1, 14]).

The most important positive characteristics of the cSPH-TVD approach are the following:
– an adequate calculation of the dynamic boundaries between wet and dry beds in case of non-stationary fluxes through the strongly inhomogeneous bottom (even through the discontinuous topography);
– calculation for subcritical (with Froude number $Fr = \sqrt{u^2+v^2}/\sqrt{gH} < 1$) and supercritical ($Fr > 1$) fluxes without isolation of these zones;
– numerical scheme of CSPH-TVD is conservative, well-balanced and has the second order of accuracy for smooth solutions the first order accuracy approximation in the vicinity of breaks and fracture profiles.

Both non steady solutions and strong heterogeneity of the topography require the specific boundary conditions formulation. Ref. [15] is dedicated to the application of the boundary conditions for the same aims by using the conditions of "waterfall" type, which we adopted in the current study.

The software implementation by using parallel technologies on graphics accelerators is presented in [12]. All basic calculations were performed on the GPU NVIDIA Tesla K80 [12]. We use Digital Elevation Models (DEM) with spatial resolution $\Delta x = \Delta y =$ 50 m and 25 m, which is based on combination of several geodata: ASTER GDEM 2 (Global DIGITAL Elevation Model), SRTM X-SAR (Shuttle Radar Topography Mission) and Sentinel-1 SAR data, topographic data for coastlines of the hydrological system, our GPS / GLONASS measurements. To improve the model we use the sailing directions and special numerical hydrodynamical simulations allowing us to compare our results with observational data.

2. The Optimal Dam Location. For a given Volga Hydroelectric Power Station hydrograph $Q_0(t)$ we can calculate the water volume entering the Akhtuba during the spring flood (see Fig. 1):

$$V_A = \int\limits_A^B \int\limits_{t_{Qs}}^{t_{Qe}} H(x,y,t)\,(u\cdot n_x + v\cdot n_y)\; dt\, d\ell\,, \tag{5}$$

where the unit vector $\vec{n} = (n_x, n_y)$ is a normal to the Akhtuba river section line (A, B) (see. Fig. 3 b), t_{Qs} and t_{Qe} are the water release at beginning and final time, respectively (see. Fig. 2). Functions $H(x,y,t)$, $u(x,y,t)$ and $v(x,y,t)$ are calculated by using the hydrodynamical model (1) – (3). We set up the dam of length L_d at the Volga riverbed close to the Akhtuba springhead at the point (x_d, y_d) (dam's center) which is perpendicular to the coastlines. The dam affects on the flow structure and on the value of V_A. For a given L_d and $Q_0(t)$ we have function $V_A(x_d, y_d)$. We calculate the water-retaining dam optimal location for the specific riverbed area S_A according to the following condition:

$$V_A^{(\max)} = \max_{(x_d,y_d)\in S_A} V_A(x_d, y_d)\,, \tag{6}$$

S_A is part of the Volga riverbed near the beginning of the Akhtuba (about 7 km downstream and 5 km upstream).

In Fig. 3 we show the $V_A^{(\max)}$ search procedure by using the gradient descent method:

$$\vec{r}_d^{\,(k+1)} = \vec{r}_d^{\,(k)} + \lambda^{(k)}\, \mathrm{grad}(V_A(\vec{r}_d^{\,(k)}))\,. \tag{7}$$

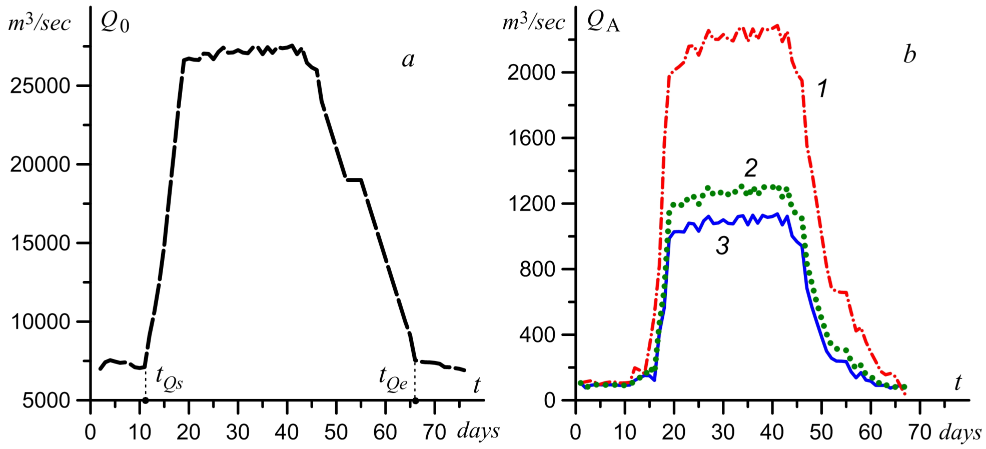

Fig. 2. The spring hydrograph $Q_0(t)$ of 2016 through the Volga Hydroelectric Power Station dam adopted in the model (a). The hydrographs of Akhtuba Q_A at different dam positions (b): *1* – the best dam position; *2* – without dam; *3* – the worst dam position

Finite difference approximation for the gradient calculation is used in the following form:

$$\operatorname{grad}(V_A(\vec{r}_d)) \simeq \left\{ \frac{V_A(x_d+\delta x, y_d) - V_A(x_d, y_d)}{\delta x}; \frac{V_A(x_d, y_d+\delta y) - V_A(x_d, y_d)}{\delta y} \right\} \qquad (8)$$

on the meshgrid $x_{i+1} = x_i + \Delta x$, $y_{j+1} = y_j + \Delta y$. Our test numerical simulations demonstrated that the relations $\delta x = \delta y = 2\Delta x = 2\Delta y$ are reasonable choice.

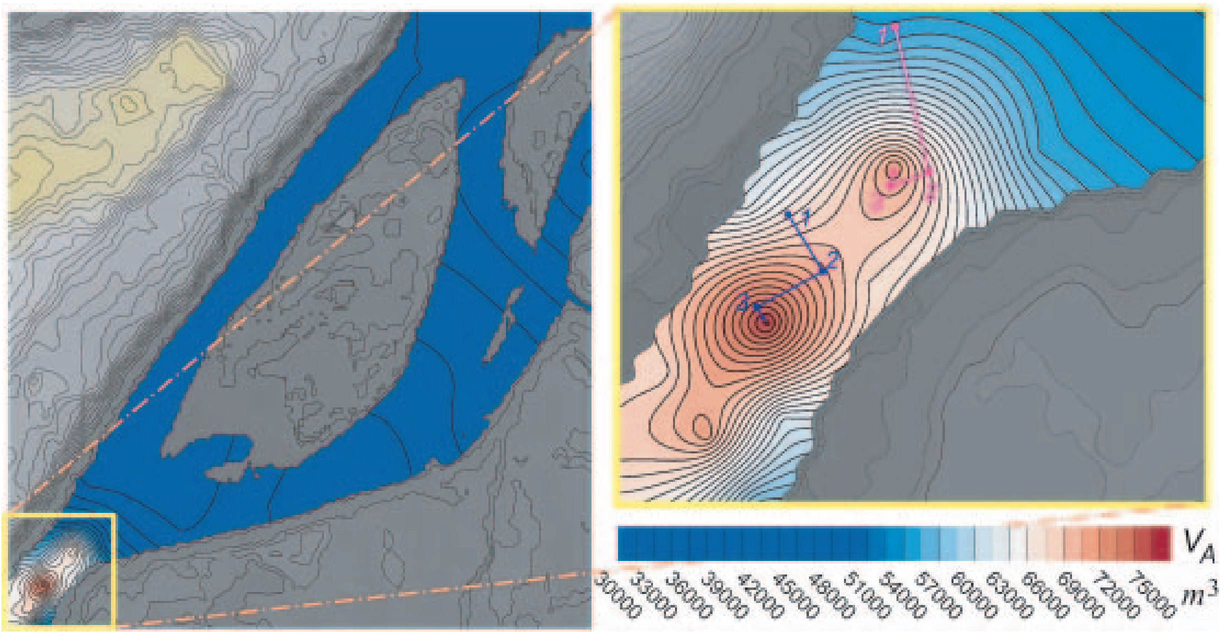

Fig. 3. The results of numerical hydrodynamical simulations for the function $V_A(x_D, y_D)$ and examples of the iterative procedure for the calculation of $V_A^{(\max)}$ in case of $L_d = 300\,\text{m}$

The symbols in Fig. 3 a indicate the positions of the iterative procedure (7), (8). The choice of λ_k parameter and the convergence of calculations are achieved by using the steepest descent method.

3. Results and Discussion. By means of numerical simulations we have studied the problem of the water-retaining dam location optimization at the Volga riverbed with the aim to increase the water amount at the left sleeve of the river Akhtuba. We summarize our results as following. There are several dam locations providing the appearance of the water volume local maximum V_A in the Akhtuba during the spring flood. The positions $(x_{dm}^{(\max)}, y_{dm}^{(\max)})$ $(m = 1, 2, 3)$ are located about 6 km downstream from the Akhtuba's beginning, that caused by the Volga riverbed structure at the area due to the large Volga width nearby the Akhtuba and large island (see Fig. 1). The best solution is $V_A^{(\max)} = 75000\,\mathrm{m}^3$. As a result, we can achieve a factor of 1,6 for maximum value of the hydrograph $Q_A(t)$ (see Fig. 2) and almost factor of 2 for V_A from $V_{A0} = 43000\,\mathrm{m}^3$ up to $V_{A0} = 75000\,\mathrm{m}^3$ in comparison to the absence of the extra dam ($V_{A0} = 43000\,\mathrm{m}^3$).

For a fixed set of free parameters any deviation of the dam orientation from the perpendicular relatively to the coastlines reduces the value V_A. The optimum location slightly depends on the dam size. It should be noted that there are some dam positions that determine the value of V_A which is smaller than in the case of absence of the water-retaining dam. Our analysis is robust to small scale perturbations of the digital topography. A small variation of the function $b(x, y)$ conserve the approximate optimal solution $(x_d^{(\max)}, y_d^{(\max)})$, but the issue still requires further investigation.

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КОМПЬЮТЕРНОЕ МОДЕЛИРОВАНИЕ ГИДРОЛОГИЧЕСКОГО РЕЖИМА ВОЛГИ: ЗАДАЧА ОПТИМАЛЬНОГО РАСПОЛОЖЕНИЯ ВОДОНАПОРНОЙ ДАМБЫ

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Рассмотрена задача оптимального расположения специальной дамбы в русле Волги в зоне начала ее левого рукава Ахтуба (7 км южнее плотины Волжской ГЭС). Новая водоподпорная дамба способна решить ключевую проблему Волго-Ахтубинской поймы, связанную с недостаточным объемом воды в период весеннего паводка из-за зарегулированности Нижней Волги. В основе математической модели расчета динамики воды лежит численное интегрирование нестационарных уравнений Сен-Венана для реального рельефа местности северной части Волго-Ахтубинской поймы. Результатом такого моделирования является расчет объема воды V_A, проходящий за весенний период в Ахтубу для определенного потока воды через гидроэлектростанцию (так называемый гидрограф характеризует объем воды, протекающей в единицу времени). Варьируя положение водоподпорной дамбы x_d, y_d, мы получаем различную структуру течения на территории в период паводка и различные значения $V_A(x_d, y_d)$. Использование метода градиентного спуска дает координаты дамбы с максимальным значением V_A. Такой подход к выбору положения дамбы позволяет рассчитать наиболее оптимальное решение, так что величина V_A увеличивается примерно в 2 раза. Наше исследование демонстрирует большие возможности математического моделирования при проектировании такого рода гидросооружений.

Ключевые слова: гидродинамическое моделирование; уравнение Сен-Венана; численная модель; оптимизация; гидрология.

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