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ACCELERATION OF SUMMATION OVER SEGMENTS USING THE FAST HOUGH TRANSFORMATION PYRAMID

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In this paper, we propose an algorithm for fast approximate calculation of the sums over arbitrary segments given by a pair of pixels in the image. Using the results of intermediate calculations of the fast Hough transform, the proposed algorithm allows to calculate the sum over arbitrary line segment with a logarithmic complexity depending on the linear size of the original image (also called fast discrete Radon transform or Brady transform). In this approach, the key element of the algorithm is the search for the dyadic straight line passing through two given pixels. We propose an algorithm for solving this problem that does not degrade the general asymptotics. We prove the correctness of the algorithm and describe a generalization of this approach to the three-dimensional case for segments of straight lines and of planes.

Keywords: search for segments; fast Hough transformation; discrete Radon transformation; Brady algorithm; fast discrete Radon transformation; dyadic pattern; beamlet pyramid.

Dedicated to Professor V.L. Arlazarov
in honour of his anniversary.

Introduction

In the paper [1], D.L. Donoho and coauthors propose an approach to multiscale image analysis called the *beamlet analysis*. The approach allows to construct detectors of objects (segments having unknown position, length and orientation; circles having unknown position and diameter, etc.) on highly noisy images [2]. From a practical point of view, the proposed approach has a drawback, since a complex pre-calculation is necessary. Indeed, for an input image, all beamlet analysis algorithms assume that the *beamlet dictionary* is pre-calculated as a set of segments having various orientations, while the position and length of the segments are chosen in a special way. However, the authors do not discuss the computational complexity of constructing such a dictionary and do not propose algorithms to calculate the dictionary. Therefore, it is difficult to estimate the practical applicability of the approach as a whole. The paper is devoted to the issues of the pre-calculation complexity, as well as the fast finding of the sums over arbitrary segments in the image as a whole.

Let us describe the approach proposed by D.L. Donoho and coauthors in more detail. The approach involves construction of a dyadically organized beamlet dictionary. The authors consider an image as a function of two variables defined on a discrete lattice in the square $[0,1]^2$ with the discretization step $1/n$, where $n = 2^p,\ p \in \mathbb{N}$, and the length of the segment is defined as a number of pixels formed the segment. Then the *dyadic square*

$S(k_1, k_2, q)$ is the set of points $[k_1/2^q, (k_1+1)/2^q] \times [k_2/2^q, (k_2+1)/2^q]$ on the plane. Here q defines the *level* of the dyadic square, and (k_1, k_2) define the position of the square, where $0 \le q \le p$ and $0 \le k_1, k_2 \le 2^q - 1$. In particular, the whole image is the dyadic square $S(0, 0, 0)$ of the level 0, and an arbitrary pixel having the coordinates (i, j) is the dyadic square $S(i, j, p)$ of the level p. A *beamlet* is a segment having arbitrary orientation whose ends belong to the sides of the same dyadic square. The coordinates of the beamlet ends are also given with a certain discretization, and the discretization quantum can be taken to be equal to or smaller than the size of a pixel. In the present paper, we consider only the first case, while the subpixel discretization is not of our interest.

By a *beamlet transformation* we mean an integration of an image over all possible beamlets in the dictionary. For an image of the size $n \times n$ pixels, the beamlet transformation allow to construct a *beamlet pyramid* containing $\Theta(n^2 \log n)$ values. The beamlet pyramid is the basis for the approach to image analysis proposed by D.L Donoho and coauthors [2], and the issue of calculation of the beamlet pyramid is considered, in particular, in the present paper.

The most obvious way to calculate the sum over pixels of a segment is to sum the values of these pixels. Further, we refer to an algorithm based on this obvious way as the "naive" algorithm. If segments have a significant intersection (such a case is typical for the beamlet dictionary), then the "naive" algorithm is ineffective, since the common sub-sums are calculated many times. For example, the "naive" algorithm for calculating the beamlet transformation requires $\Theta(n^3)$ operations, which is too much to use in real time and on mobile platforms. However, D.L Donoho and coauthors refuse to consider the issue of fast calculation of the beamlet transformation and assume such a result to be already known for a given image. At the same time, the authors list several studies aimed at fast calculation of the discrete Radon transformation, which is similar to the beamlet transformation. One of these approaches is the algorithm of the fast discrete Radon transformation. The algorithm was proposed by P. Brady [3] and is also known as the *fast Hough transformation* (FHT) [4]. Nowadays, the algorithm is studied in detail. In $\Theta(n^2 \log n)$ operations, the algorithm allows to calculate accurately the total Hough-image (i.e. an image, each pixel of which contains the sum over the pattern, the parameters of which are usually given by the coordinates of this pixel) over the dyadic patterns (further, we refer to dyadic patterns as *dyadic straight lines*). The dyadic straight lines well approximate straight lines on the plane, since the approximation error along the ordinate axis is $\frac{\log_2 n}{6}$ for straight lines with the slope $k < 1$ in the parametrization by an angular coefficient [5]. Moreover, it is proved that the asymptotic complexity of the FHT algorithm cannot be improved [6].

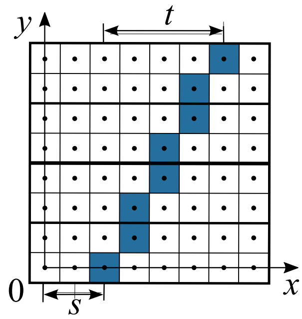

Fig. 1. The (s, t)-parameterization of a predominantly horizontal pattern with a downward slope $(t > 0)$

In the FHT algorithm, the sum over a dyadic pattern is composed of two sums over halves of the pattern, which are also dyadic patterns, while the sums over the halves are also composed of the sums over their halves, etc. Therefore, if intermediate results are saved during the process of calculating the FHT, then a *FHT pyramid* is formed, which is largely similar to a beamlet pyramid. Saving intermediate results does not increase asymptotic complexity of the FHT algorithm, therefore, the FHT pyramid can be constructed in $\Theta(n^2 \log n)$ operations.

In this paper, we propose an algorithm for fast (in $\Theta(\log n)$ operations) finding of the sum over an arbitrary (given by a pair of points) segment of a dyadic pattern in an image. This algorithm is based on the FHT-pyramid and can be used as a method for calculating the *fast beamlet transformation* (approximation of the beamlet transformation summing over dyadic patterns). The asymptotic complexity of the fast beamlet transformation is $\Theta(n^2 \log^2 n)$ operations. The proposed algorithm is also of independent value in problems that require to calculate numerous sums over segments in an image, but do not require such a high-level theory as the beamlet analysis [7]. At the same time, if it is necessary to find only a summation over segments having a certain range of slope angles, then the volume of the pre-calculation can be reduced by performing a partial FHT and calculating the pyramid only for the partial FHT [8].

For three-dimensional images, two generalizations of the Radon transformation are known: the three-dimensional Radon transformation (summations over straight lines in the three-dimensional space) and the John transformation (summations over planes). For both generalizations, the algorithm for fast summation over dyadic patterns is known: the three-dimensional fast Hough transformation is used over straight lines and planes, respectively [9]. In this paper, we propose an algorithm for fast summation over segments of dyadic straight lines in a three-dimensional image and over segments of dyadic planes of a special type.

1. Fast Finding of the Sum Over an Arbitrary Segment

For brevity, we consider only square images of the size $n \times n$ pixels, where $n = 2^p$. Following the paper [9], we use the (s,t)-parameterization of straight lines in the image. To this end, first of all, we divide all the straight lines into two classes: *predominantly vertical* straight lines (i.e. vertical straight lines and straight lines with $|k| \geq 1$ in the parameterization by the angular coefficient) and *predominantly horizontal* straight lines (i.e. straight lines with $|k| \leq 1$). Therefore, we refer straight lines with $k = 1$,

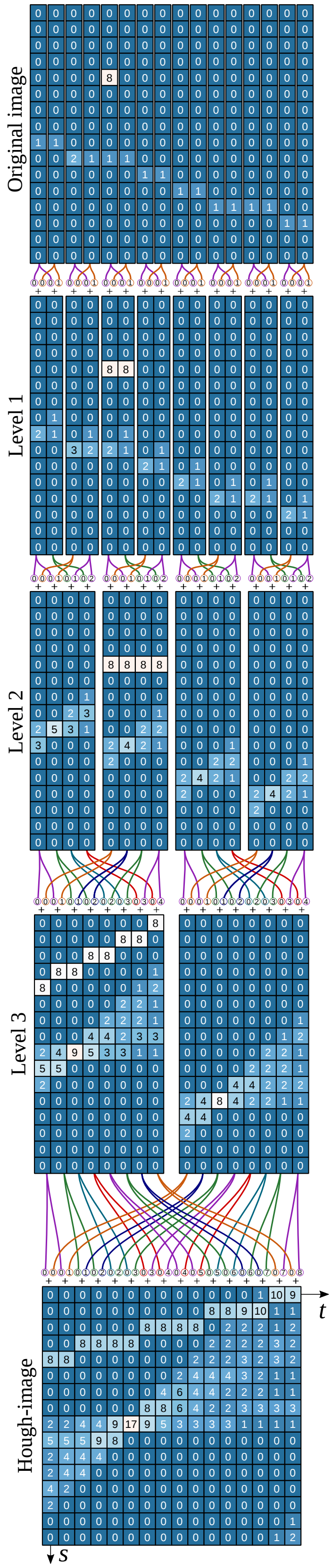

Fig. 2. FHT-pyramid for an image of the size 16×16

i.e. inclined at an angle 45°, to both classes. Note that the FHT for predominantly horizontal straight lines is also suitable for predominantly vertical ones. Indeed, it is enough to rotate the original image by 90°. Therefore, further we consider only predominantly horizontal straight lines. Such a straight line can be given by the parameters s and t of the coordinates of two points on the vertical boundaries of the image: $(0, s)$ and $(n-1, s+t)$, where $t \in [-n+1; n-1]$ and $s \in [-n+1; n-1]$, see Fig. 1. For brevity, we consider only the case of $t \geq 0$, i.e. *predominantly-horizontal patterns inclined downward* and the Hough-image for such patterns.

A Hough-image is represented as an image of the size $n \times n$, whose each point (s, t) contains the sum over the pattern with such parameters. In this paper, we are interested not only in the algorithm itself, but also in the intermediate results of the algorithm implementation, which can be represented as $(\log_2 n + 1)$ images (levels) of the size $n \times n$ forming a FHT-pyramid.

1.1. FHT-Pyramid

Taking into account the assumptions and the definitions given above, we describe the FHT-pyramid and the method of its calculation. The zero level of the FHT-pyramid is the original image, and the last level is the Hough-image containing sums over dyadic straight lines of the length $n = 2^p$. Similarly to the description of the beamlet pyramid level, we describe the q-th level of the FHT-pyramid. To this end, divide the original image into vertical stripes $[k \cdot 2^q, (k+1) \cdot 2^q - 1] \times [0, n-1]$, where k denotes the number of the stripe, $k \in [0, 2^{p-q} - 1]$. For each stripe, the q-th level of the FHT-pyramid contains the sums over all possible patterns of the stripe that have the length 2^q and the parameters $s \in [0, n-1]$ and $t \in [0, 2^q - 1]$. The number of such patterns is equal to $n \times 2^q$ for each strip. Therefore, the q-th level of the FHT-pyramid requires as much memory as the original image. Fig. 2 shows an example of the algorithm implementation and the formation of all levels of the FHT-pyramid.

1.2. Finding of the Sum Over a Segment of a Known Dyadic Straight Line

Consider a fast way to calculate the sum over any segment in the image using the FHT-pyramid [10].

Suppose that it is necessary to find the sum over the segment $B'C$ of a predominantly horizontal inclined downward dyadic straight line AD, whose parameters (s, t) are known. Let us find the sum as the difference between the sums over the segments AC and AB.

Denote the length of the segment AB by l. Let us find the sum over AB as the sum of the minimum number of already calculated sums saved in the FHT-pyramid, see Fig. 3.

Suppose that the binary expansion of the length of AB has the form $l = \sum_{j=0}^{p} l_j 2^j$. Then the sum over the segment AB is calculated by the formula

$$S^{s,t}(l) = \sum_{q=0}^{p} \left[l_q \cdot H^{s,t}\Big(q, \sum_{j=q+1}^{p} l_j \cdot 2^{j-q}\Big) \right],$$

where $H^{s,t}(q, k)$ denotes the sum over the pattern of the length 2^q in the k-th stripe of the q-th level of the FHT-pyramid for a straight line with the parameters (s, t).

The internal sum does not require a complete calculation at each step, since the sum is obtained from the previous one in constant time. Therefore, the complexity of the algorithm is proportional to the number of terms in the external sum, i.e. is equal to $\Theta(\log n)$. Similarly, the sum over the segment AC is calculated.

For the case of dyadic straight lines in a three-dimensional image, note that the described algorithm for finding of the sums is almost identical, except that the search is performed in a four-dimensional FHT-pyramid, which does not affect the complexity of the search due to dyadic organization of the FHT-pyramid.

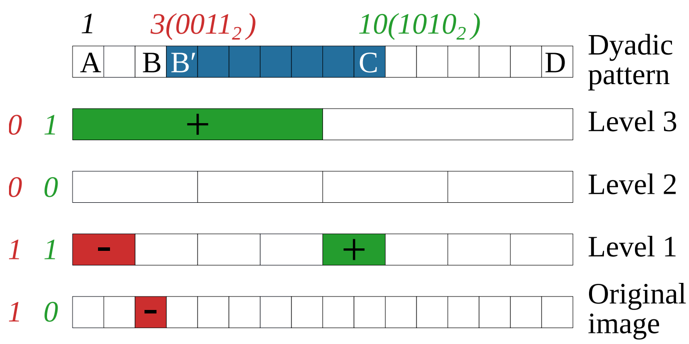

Fig. 3. Calculation of the sum over the segment $B'C$ of the straight line AD as the difference between the sums over the segments AC and AB

1.3. Finding of a Dyadic Pattern

The paper [9] proves that at least one dyadic pattern passes through any two pixels, but such a pattern can be not unique. Let us solve the problem on finding of the parameters (s,t) of any one of the patterns passing through the given points (x_1,y_1) and (x_2,y_2). Without loss of generality, we assume that $x_2 \geq x_1$. Since, as stated above, we consider only predominantly horizontal patterns inclined downwards, then $y_2 \geq y_1$ and $y_2 - y_1 \leq x_2 - x_1$.

Suppose that the binary expansion of the parameter t of the desired pattern has the form $t = \sum_{i=0}^{p-1} t_i 2^i$. Then the pattern can be given by the following equation [5]:

$$y = D^{s,t}(x) = s + \sum_{r=0}^{p-1} t_r \left[\frac{2^r x}{2^p - 1}\right], \tag{1}$$

where $[\,\cdot\,]$ denotes rounding to the nearest integer. Expression (1) shows that translation of any dyadic pattern can be represented as the sum of subset translations of the basic dyadic patterns of the form

$$D_r(x) = \left[\frac{2^r x}{2^p - 1}\right]. \tag{2}$$

Substitute the coordinates of the points (x_1,y_1) and (x_2,y_2) into expression (1) for the pattern and subtract the second equation from the first one:

$$y_2 - y_1 = D^{s,t}(x_2) - D^{s,t}(x_1) = \sum_{r=0}^{p-1} t_r (D_r(x_2) - D_r(x_1)).$$

Denote $\Delta y = y_2 - y_1$. Let $\delta_i^{x_1,x_2} = D_i(x_2) - D_i(x_1)$ be the number of shifts of the i-th basic dyadic pattern in the stripe (x_1,x_2), then the equation takes the form

$$\Delta y = \sum_{r=0}^{p-1} t_r \delta_r^{x_1,x_2}.$$

Therefore, the problem on finding of a slope is reduced to the following problem: to restore the bits t_0, t_1, ..., t_{p-1} for the known Δy and $\delta_0^{x_1,x_2}$, $\delta_1^{x_1,x_2}$, ..., $\delta_{p-1}^{x_1,x_2}$.

To this end, apply the "greedy" **algorithm**. First, consider all t_i to be zero. Since $\delta_i^{x_1,x_2} \geq \delta_j^{x_1,x_2}$ for $i > j$, we go through shifts from the large number to less one, that is, go from the larger level to zero one. If $\Delta y \geq \delta_i^{x_1,x_2}$, then consider t_i to be equal to 1 and reduce Δy by $\delta_i^{x_1,x_2}$. Repeat the procedure until Δy is not equal to zero (we prove that $\Delta y = 0$ at some moment, i.e. that the algorithm is correct, below).

Use the value of the parameter t to find the parameter s of the dyadic pattern as

$$s = D(x_1, t) - \sum_{r=0}^{p-1} t_r D_r(x_1).$$

Since $\#\{\delta_i^{x_1,x_2}\} = p = \log_2 n$, then the complexity of the algorithm is $\Theta(\log n)$ operations.

1.3.1. Correctness of the Algorithm

Theorem 1. *$D_j(x) - 1 \leq \sum\limits_{i<j} D_i(x) \leq D_j(x)$ $\forall x \in \overline{0, n-1}$ and $\forall j \in \overline{0, p-1}$.*

Proof. For $j \in \overline{0, p-1}$, consider the sets J_j formed by the coordinates of the shift pixels of the basis dyadic pattern with slopes 2^j, i.e. those for which the ordinate is greater than the ordinate of the previous one: $D_j(x) = D_j(x-1) + 1$.

Since $|J_{j+1}| = 2|J_j|$, then, similar to a binary tree, where the number of leaves is greater by unity than the number of other nodes, we obtain that $J_j = \sum\limits_{i<j} J_i + 1$. According to formula (2), the distance between two adjacent shift pixels is the same for the entire width of an image, and $J_i \cap J_j = \varnothing, \forall i, j \in [0, p-1], i \neq j$.

From the properties given above, it follows that the shift pixels of the set J_j alternate with the shift pixels of all other sets J_i (for $i < j$), and the first shift pixel belongs to the set J_j.

Therefore, $\forall x \in [0, n-1]$, the number of boundary pixels of $D_j(x)$ is either equal to or greater by unity than $\sum\limits_{i<j} D_i(x)$.

□

Corollary 1.

$$\sum_{i<j} \delta_i^{x_1,x_2} - 1 \leq \delta_j^{x_1,x_2} \leq \sum_{i<j} \delta_i^{x_1,x_2} + 1 \tag{3}$$

$\forall j \in \overline{0, p-1}$ and $\forall x_1, x_2 \in [0, n-1]$ such that $x_1 > x_2$.

Proof. Rewrite Theorem 1 as $\sum\limits_{i<j} D_i(x) \leq D_j(x) \leq \sum\limits_{i<j} D_i(x) + 1$. Apply this pair of inequalities to the points x_1 and x_2 and pairwise add the inequalities taking into account the signs.

□

Theorem 2. *If the numbers $\delta_0^{x_1,x_2}, \delta_1^{x_1,x_2}, ..., \delta_k^{x_1,x_2}$ satisfy condition (3), then the sums of subset translations of their every possible subsets cover the range of values from 0 to $\sum\limits_{i=0}^{k} \delta_i^{x_1,x_2}$.*

Proof. Let us prove by induction. First, we show that the statement is true for $i = 0$. Formula (2) implies that $\delta_0^{x_1,x_2} = [\frac{x_2}{n-1}] - [\frac{x_1}{n-1}] \geq 0$, since $x_2 > x_1$ by condition, i.e.

$$0 \leq \delta_0^{x_1,x_2} \leq 1.$$

Second, suppose that the statement is true for $i = k - 1$, that is, we can obtain all sums from 0 to $\sum_{i=0}^{k-1} \delta_i^{x_1,x_2}$ for the given numbers $\delta_0^{x_1,x_2}, \delta_1^{x_1,x_2}, ..., \delta_{k-1}^{x_1,x_2}$.

Finally, show that the statement is true for $i = k$. Inequality (3) is satisfied for $\delta_k^{x_1,x_2}$. In other words, the difference between $\delta_k^{x_1,x_2}$ and $\sum_{i=0}^{k-1} \delta_i^{x_1,x_2}$ is no more than 1. Therefore, we can add $\delta_k^{x_1,x_2}$ to every possible sum of the numbers $\delta_0^{x_1,x_2}, \delta_1^{x_1,x_2}, ..., \delta_{k-1}^{x_1,x_2}$ to obtain all sums from 0 to $\sum_{i=0}^{k} \delta_i^{x_1,x_2}$. This completes the proof.

□

Therefore, if $\Delta y \leq \sum_{i=0}^{p-1} \delta_i^{x_1,x_2}$, then there exists a dyadic pattern passing through two given pixels.

Theorem 3. *The proposed algorithm to find a dyadic pattern always converges to a solution.*

Proof. Since we go through $\delta_k^{x_1,x_2}$ from $k = p - 1$ to $k = 0$, then inequality (3) is satisfied for all $\delta_0^{x_1,x_2}, \delta_1^{x_1,x_2} ... \delta_{k-1}^{x_1,x_2}$ at each stage, therefore, Theorem 2 is true for $\delta_0^{x_1,x_2}, \delta_1^{x_1,x_2} ... \delta_{k-1}^{x_1,x_2}$. Hence, the problem has a solution at each stage. Therefore, we always obtain a solution to the problem.

□

The proposed algorithm and its properties are directly generalized to the case of straight lines in large dimensions. To this end, it is enough to solve the problem for projections of a straight line of the length n on the coordinate planes sequentially.

2. Fast Finding of the Sum Over a Segment of a Plane

Following the paper [9], we introduce the typification of planes in a three-dimensional image. A plane is said to be *predominantly perpendicular to a coordinate axis*, if an angle between the normal to the plane and this coordinate axis is smaller than each of angles between the normal and other coordinate axis. We consider only the planes that are predominantly perpendicular to the coordinate axis OX. Such planes are divided into four types depending on the slopes, see Fig. 4. Without loss of generality, we assume that the considered planes are of the type I.

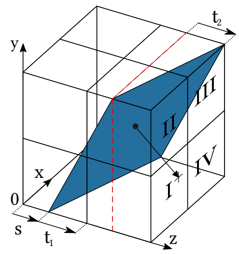

Fig. 4. Typification of planes in the space given in the paper [9]

Let $EFGH$ be a dyadic plane in which it is necessary to find the sum over a segment. Similarly to the previous section, we consider the parameters (s, t_1, t_2) of the dyadic plane $EFGH$ to be already found. As the segments, we consider the parallelograms, whose sides are parallel to the faces of the image. For example, see the parallelogram $ABCD$ in Fig. 5.

Let us briefly describe the procedure of pre-calculation. According to the paper [9], the algorithm of a three-dimensional FHT for planes is as follows. First, apply a two-dimensional FHT to each horizontal layer of a three-dimensional image (such an operation is called the *horizontal FHT*). Then, in the resulting image, apply a FHT to each vertical

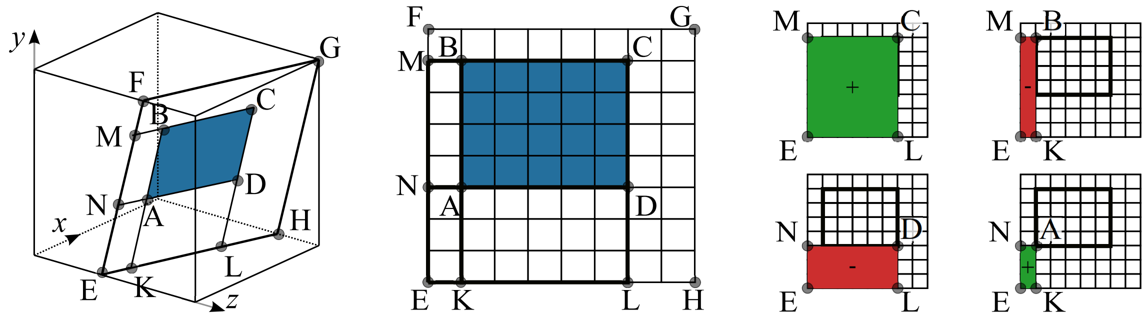

Fig. 5. The dyadic plane $EFGH$ containing the segment $ABCD$, and their projections to the coordinate plane OYX

plane (such an operation is called the *vertical FHT*). In order to construct a three-dimensional FHT pyramid, we apply the vertical FHT (and save all intermediate values) to the two-dimensional FHT pyramids constructed in the process of applying the horizontal FHT to each horizontal layer. In other words, the three-dimensional FHT pyramid contains the sums over all rectangles of the form $2^l \times 2^m$ from the dyadic division of the Donoho-type plane.

Since the vertical FHT is applied to $n \log_2(n)$ planes, and the complexity of the two-dimensional FHT is $\Theta(n^2 \log n)$, then the described calculation of the three-dimensional FHT pyramid has the complexity $\Theta(n^3 \log^2 n)$.

The sum over the segment $ABCD$ can be calculated by the inclusion-exclusion formula

$$S_{ABCD} = S_{EMCL} - S_{EMBK} - S_{ENDL} + S_{ENAK}.$$

Note that the point E is one of the angles for each segment. Further, we refer to such segments as the *angle segments*.

Consider the angle segment $EMCL$. It is necessary to divide $EMCL$ into the segments presented in the FHT pyramid. Consider binary expansions of the width and height of a segment. Similarly to the one-dimensional case, divide the segment horizontally into $\log_2 n$ vertical stripes and vertically into at most $\log_2 n$ horizontal stripes.

Intersection of these stripes contains no more than $\log_2^2(n)$ segments presented in the three-dimensional FHT pyramid, see Fig. 6. Therefore, the complexity of calculating the sum over an arbitrary segment is $\Theta(\log^2 n)$ operations.

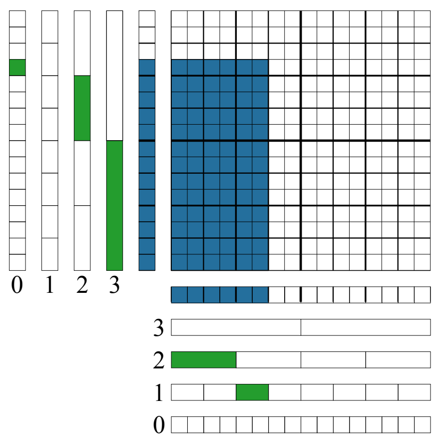

Fig. 6. Algorithm to find a segment of a plane taking into account the previously calculated three-dimensional FHT pyramid

Discussions

The algorithms similar to FHT have an important practical property: in order to calculate such an algorithm, it is enough to use the addition operation only. This fact is essential for the calculation complexity. Algorithms having good computational characteristics are especially in demand to create modern neural network detectors. For example, the paper [11] proposes an effective method to find the vanishing point of an image using a convolutional neural network containing FHT layers capable to detect straight lines in the image. In the paper [11], the use of FHT allows to reduce significantly the size of the neural network under the same quality of detection. By analogy, we can assume that, in the problems on detection of segments of the image, effective convolutional neural networks include layers that calculate the FHT pyramid.

Also, the developed solutions can be useful to create computer vision systems based on the algorithms of segment detection, for example, in combination with the methods proposed by D.L. Donoho and coauthors.

On the other hand, a number of nontrivial and interesting mathematical problems arise when the FHT is generalized to the cases of higher dimensions. In particular, the existence of a dyadic plane passing through any three given voxels is not proved yet, to say nothing of generalizations of these statements to the cases of higher dimensions.

The study and development of the algorithms similar to FHT has good potential both for use in various image-based systems of navigation and localization and for new mathematical problems.

Conclusion

In this paper, we present an algorithm for the fast search for segments of straight lines and segments of planes in the two-dimensional and three-dimensional cases using the corresponding fast Hough transformation as a pre-calculation. Also, we propose a method to find the parameters of a dyadic straight line passing through arbitrary two pixels of an image. In order to continue the development of this work, we plan to study the issue of proving the theorem that at least one dyadic plane passes through any three voxels, as well as to investigate the possibility to calculate fast the sum over an arbitrary triangular segment in a three-dimensional image.

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УСКОРЕНИЕ СУММАЦИИ ПО ОТРЕЗКАМ С ИСПОЛЬЗОВАНИЕМ ПИРАМИДЫ БЫСТРОГО ПРЕОБРАЗОВАНИЯ ХАФА

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В работе предложен алгоритм быстрого приближенного вычисления на изображении сумм по произвольным отрезкам, задаваемым парой пикселей. Используя результаты промежуточных вычислений быстрого преобразования Хафа, предложенный алгоритм позволяет рассчитать сумму по произвольному отрезку линии с логарифмической сложностью, зависящей от линейного размера исходного изображения. Предподсчет реализован как модификация алгоритма Брейди быстрой суммации по диадическим аппроксимациям прямых. При таком подходе ключевым элементом алгоритма является поиск диадической прямой, проходящей через два данных пикселя. В работе предложен алгоритм решения этой задачи, не ухудшающий общую асимптотику, для него доказана корректность. Также в работе описывается обобщение этого подхода на трехмерный случай для отрезков и для сегментов плоскостей.

Ключевые слова: поиск отрезков; быстрое преобразование Хафа; дискретное преобразование Радона; алгоритм Брейди; быстрое дискретное преобразование Радона; диадический паттерн; бимлет-пирамида.

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