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MATHEMATICAL MODELLING OF INDUSTRIAL EQUIPMENT OPERATION BASED ON MARKOV PROCESSES

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The existence of an almost unlimited number of methods for evaluating the productivity of industrial equipment contributes to the uncertainty of choosing the most effective approach. At the same time, the presence of many possible states of equipment (from working to repair and other downtime) complicates the problem of modelling the operation of such a system. The problem of modelling does not lose its relevance, first of all, for large industrial companies. The article presents the methodological features of modelling the operation of industrial equipment based on the Markov method. This approach is used as a base for estimating the probabilities of equipment transitions between states, as well as for predicting the final state of operation of such a system. In terms of practical application, we consider an example of the functioning of the same type of industrial equipment in the framework of three possible states (functional, broken, and also in the mode of forced repair). Based on the results of calculations, we carry out assessment of the reality of the state transitions of equipment, designed rate of these transitions, as well as the predicted level of productivity equipment system after the period "t". The reliability of the research results is confirmed by their practical implementation. The obtained results are recommended to be used by the management and analysts of industrial companies in the process of making operational decisions and in the development of equipment repair strategies.

Keywords: Markov method; mathematical modelling; theory of probability; prediction; industrial equipment; equipment states; states transitions.

Introduction

The relevance of the problem of modelling the effective operation of industrial equipment for large companies is due to a number of significant reasons. The first reason is mandatory to implement the company production plans. The second one is the improving the reliability of equipment operation, improving its repair and maintenance systems [1, 2]. Thirdly, the need for accurate forecasting of equipment repair and maintenance costs. Together, these reasons are integral elements of the strategy of an industrial company development. Currently, this issue is updated due to the lack of uniform clear guidelines for the maintenance of equipment at industrial enterprises. The authors divide the solution of this problem into three stages.

1. Theoretical Foundations of Markov Modelling of Equipment Operation

Markov analysis is based on the concept of possible states of equipment and the transition between these states in time, assuming a constant probability of such transitions [3]. Based on the initial data on the functioning of the system under study, we construct a Markov transition matrix (Table), as well as its graphical representation in the form of a diagram of states and transitions (Figure). They describe the probabilities of transitions between different states of equipment and model aspects of the reliability of the system behaviour over time.

Markov analysis is applicable in a situation where the future state of the system depends only on its current state. This method is usually used in industry companies to analyse maintainable systems that can operate in many modes.

The *input data* of the Markov analysis are [4]:

– a list of different system states (e.g. full or partial operation, failure, overhaul, scheduled maintenance, etc.);

– an accurate understanding of the possible transitions that need to be modeled;

– the rate of transition from one state to another, usually in the form of the transition probability (in the case of discrete analysis), the failure (λ) or recovery (μ) rate (in the case of continuous analysis).

The *output data* are the values of the probabilities that the system is in various states, as well as the modelling of the system state after a specific period [4].

2. Estimation of Probabilities of Industrial Equipment Transitions between States Based on the Markov Method

In this section, we consider an example of the operation of the same type of industrial equipment for the case of discrete analysis. According to the initial data, there are three possible states of equipment:

– S_1 is a working state;

– S_2 is a broken state;

– S_3 is a state to be on a forced (unscheduled) repair.

The initial characteristic of this system is as follows: there are 1300, 40 and 12 pieces of equipment in the state S_1, S_2 and S_3, respectively.

The matrix of the transitions from the initial state to the final state is presented in Table. The graphical interpretation of Table is shown in Figure. The indicator "t" means the number of periods when the object is in an unchanged state, and the unknowns X_1, X_2, X_3, X_4 are the probabilities of equipment transitions to different states.

Table

Matrix of the equipment states transitions

	Final state			
Initial state		S_1	S_2	S_3
	S_1	0,95	X_2	X_3
	S_2	0	t^{-2}	X_4
	S_3	X_1	0,04	0,27

The values shown in Table and Figure interpret the probabilities of the equipment transition from the initial state to the final state, for example:

– 0,95 means that the probability that the equipment will remain in the working state S_1 is 95%;

– 0 means that there is no possibility that the equipment will immediately move from the broken state S_2 to the working state S_1 without a prior repair;

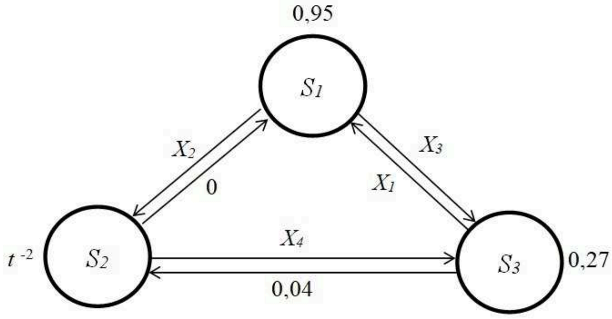

Diagram of equipment states and transitions

– t^{-2} is a dependent variable that is equal to the probability of finding the equipment in the broken state S_2, etc.

Initially, the estimation of all unknown equipment transition probabilities is based on the principles of probability theory. So, for each initial state, the sum of all the probabilities of equipment transitions to the final state is equal to one. Therefore, we construct the system

$$\begin{cases} 0,95 + X_2 + X_3 = 1, \\ 0 + t^{-2} + X_4 = 1, \\ X_1 + 0,04 + 0,27 = 1. \end{cases} \tag{1}$$

As a result, $X_1 = 0{,}69$, $X_4 = (1 - t^{-2})$, the transition probability of X_4 is dependent on the variable "t". The transitions of equipment X_2 and X_3 between states are interdependent, as shown in the equation

$$X_2 + X_3 = 0,05. \tag{2}$$

Therefore, to quantify the probabilities of these transitions, it is necessary to investigate their reality:

– X_2 is the probability of transition of the equipment from the operating mode to the broken state;

– X_3 is the probability of transition of the equipment from the operating mode to the repair state.

However, the equipment can not immediately move from a working state to a forced repair, since for this it must initially break down. Therefore, the probability of the transition X_3 is zero, hence the probability of the transition X_2 is 0,05.

3. Prediction of Final State of Operation of Industrial Equipment after Period "t" by Markov Method

To predict the final state of the equipment operation after the period "t" equal to 50 days, we construct the system of equations

$$\begin{cases} P_1 = 0,95 \cdot P_1 + 0 \cdot P_2 + X_1 \cdot P_3, \\ P_2 = X_2 \cdot P_1 + t^{-2} \cdot P_2 + 0,04 \cdot P_3, \\ P_3 = X_3 \cdot P_1 + X_4 \cdot P_2 + 0,27 \cdot P_3, \\ P_1 + P_2 + P_3 = 1, \end{cases} \tag{3}$$

where P_1, P_2, P_3 are the probabilities of finding the system in each of the states S_1, S_2 and S_3, respectively.

Suppose that the values X_1, X_2, X_3, X_4 are known, $t = 50$ days, then we specify the initial conditions:

$$\begin{cases} P_1 = 0,95 \cdot P_1 + 0 \cdot P_2 + 0,69 \cdot P_3, \\ P_2 = 0,05 \cdot P_1 + 50^{-2} \cdot P_2 + 0,04 \cdot P_3, \\ P_3 = 0 \cdot P_1 + (1 - 50^{-2}) \cdot P_2 + 0,27 \cdot P_3, \\ P_1 + P_2 + P_3 = 1. \end{cases} \tag{4}$$

System (4) has the following results:

– $P_1 = 0{,}8886$, or 88,86%;

– $P_2 = 0{,}047$, or 4,7%;

– $P_3 = 0{,}0644$, or 6,44%.

This means that, after 50 days, this system of equipment is in working state, broken state, and in the state of repair with the probability 88.86%, 4.7%, and 6,44%, respectively. Suppose that the company has exactly 1 352 pieces of equipment at its disposal, when, after 50 days, the final state of the simulated system is as follows:

– 1 201 equipment work;

– 64 is in the broken state;

– 87 is in the state of repair.

Conclusions

1. A solution to the relevant problem of modelling the operation of industrial equipment using the Markov method is proposed.

2. All possible transitions between three states of the same type of equipment are modeled and their probabilities are quantified.

3. The forecast of the final state of operation of the equipment after the period "t" is made.

4. The results obtained are recommended for use in predicting the uninterrupted operation of industrial companies equipment and developing maintenance strategies.

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МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ РАБОТЫ ПРОМЫШЛЕННОГО ОБОРУДОВАНИЯ НА ОСНОВЕ МАРКОВСКИХ ПРОЦЕССОВ

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Существование практически неограниченного числа методов оценки работоспособности промышленного оборудования способствует возникновению неопределенности выбора наиболее эффективного подхода. Одновременно с этим наличие множества возможных состояний оборудования (от рабочего до ремонтного и иного простоя) усложняет задачу моделирования работы такой системы. Задача моделирования не теряет своей актуальности, в первую очередь, для крупных промышленных компаний. В статье представлены методологические особенности моделирования работы промышленного оборудования на основе Марковского метода. Данный подход использован в качестве базового при оценке вероятностей переходов оборудования между состояниями, а также при прогнозировании конечного состояния работы подобной системы. В части практической апробации рассмотрен пример функционирования однотипного промышленного оборудования в рамках трех возможных состояний: работоспособное, сломанное, а также находящееся в режиме вынужденного ремонта. По результатам расчетов проведена оценка реальности переходов между состояниями оборудования, рассчитана скорость подобных переходов, а также спрогнозирован уровень работоспособности системы оборудования через период «t». Достоверность результатов исследования подтверждена их практической реализацией. Полученные в рамках работы результаты рекомендуется использовать менеджментом и аналитикам промышленных компаний в процессе принятия операционных решений и при разработке стратегий ремонта оборудования.

Ключевые слова: Марковский метод; математическое моделирование; теория вероятностей; прогнозирование; промышленное оборудование; состояния оборудования; переходы между состояниями.

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