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HOFF'S MODEL ON A GEOMETRIC GRAPH. SIMULATIONS

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This article studies numerically the solutions to the Showalter–Sidorov (Cauchy) initial value problem and inverse problems for the generalized Hoff model. Basing on the phase space method and a modified Galerkin method, we develop numerical algorithms to solve initial-boundary value problems and inverse problems for this model and implement them as a software bundle in the symbolic computation package Maple 15.0. Hoff's model describes the dynamics of H-beam construction. Hoff's equation, set up on each edge of a graph, describes the buckling of the H-beam.

The inverse problem consists in finding the unknown coefficients using additional measurements, which account for the change of the rate in buckling dynamics at the initial and terminal points of the beam at the initial moment. This investigation rests on the results of the theory of semi-linear Sobolev-type equations, as the initial-boundary value problem for the corresponding system of partial differential equations reduces to the abstract Showalter–Sidorov (Cauchy) problem for the Sobolev-type equation. In each example we calculate the eigenvalues and eigenfunctions of the Sturm–Liouville operator on the graph and find the solution in the form of the Galerkin sum of a few first eigenfunctions. Software enables us to graph the numerical solution and visualize the phase space of the equations of the specified problems. The results may be useful for specialists in the field of mathematical physics and mathematical modelling.

Keywords: Sobolev-type equation; Hoff's model.

Introduction

Take a finite connected oriented graph $\mathbf{G} = \mathbf{G}(\mathfrak{V};\mathfrak{E})$ with vertex set $\mathfrak{V} = \{V_i\}$ and edge set $\mathfrak{E} = \{E_j\}$. Associate to the each edge E_j two positive integers l_j, $d_j \in \mathbb{R}_+$, which have physical meaning in the context of our problem: the length and area of the cross-section of the edge.

Consider Hoff's model of the dynamics of H-beam constructions. On each edge E_j we set up the generalized Hoff equation

$$\lambda_j u_{jt} + u_{jxxt} = \alpha_{1j} u_j + \alpha_{2j} u_j^3 + ... + \alpha_{nj} u_j^{2n-1}, \quad n \in \mathbb{N}, \tag{1}$$

and at each vertex we impose the boundary conditions

$$u_j(0,t) = u_k(0,t) = u_m(l_m,t) = u_p(l_p,t), \quad E_j, E_k \in E^\alpha(V_i), E_m, E_p \in E^\omega(V_i), \tag{2}$$

$$\sum_{j:\ E_j \in E^\alpha(V_i)} d_j u_{jx}(0,t) - \sum_{m:\ E_m \in E^\omega(V_i)} d_m u_{mx}(l_m,t) = 0, \tag{3}$$

where $E^\alpha(V_i)$ and $E^\omega(V_i)$ are the sets of edges with the source and target vertex V_i respectively. The parameters $\lambda_j \in \mathbb{R}_+$ correspond to the load on beam j, and the parameters $\alpha_{sj} \in \mathbb{R}$ for $s = 1, 2, ..., n$ characterize the material of beam j. The unknown functions $u_j(x,t)$ of $x \in (0, l_j)$ and $t \in \mathbb{R}$ specify the deflection of beam j from the vertical direction.

The direct problem consists in finding the vector function $u = (u_1, u_2, ..., u_j, ...)$, each component $u_j(x,t)$ of which satisfies the continuity condition (2) and the flow balance condition (3). Besides, the components $u_j(x,t)$ must satisfy the Cauchy initial conditions

$$u_j(x, 0) = u_{j0}(x), \quad x \in (0, l_j). \tag{4}$$

or the Showalter–Sidorov initial conditions

$$\left(\lambda_j + \frac{\partial^2}{\partial x^2}\right)(u_j(x,0) - u_{j0}(x)) = 0, \quad x \in (0, l_j). \tag{5}$$

The inverse problem (in the case $n = 2$) consists in finding the solutions u_j to (1) and the unknown coefficients $\alpha_j = \alpha_{1j}$ and $\beta_j = \alpha_{2j}$ using the additional measurements

$$\alpha_j u_{0j}(0) + \beta_j u_{0j}^3(0) = \varphi_j, \quad \alpha_j u_{0j}(l_j) + \beta_j u_{0j}^3(l_j) = \psi_j, \tag{6}$$

where φ_j and ψ_j reflect the change of the rate in buckling dynamics at the endpoints of the beam at the initial moment.

The paper aims to generalize the results of [1, 2] to the case of the inverse problem for Hoff's model with the Showalter–Sidorov conditions and to develop numerical algorithms for solving the stated problems. Hoff's equation belongs to the class of semilinear Sobolev-type equations; for this reason we use here the theory of degenerate semigroups and relatively p-bounded operators. For more details concerning the theory of semigroups and Sobolev-type equations, as well as Cauchy and Showalter–Sidorov problems, see [3–7].

1. Hoff's Model

Following [1, 2], reduce (1)–(4) to the Cauchy problem $u(0) = u_0$, and (1)–(3), (5) to the Showalter–Sidorov problem $L(u(0) - u_0) = 0$ for the semilinear Sobolev-type equation $L\dot{u} = Mu + N(u)$. Consider the Hilbert space $\mathbf{L_2}(\mathbf{G})$ as well as two spaces $\mathfrak{U}$ and $\mathfrak{F}$ with the natural structure of a Banach rather than Hilbert space. Define the operators $L, M, N : \mathfrak{U} \to \mathfrak{F}$ as

$$\langle Lu, v\rangle = \sum_j d_j \int_0^{l_j} (\lambda_j u_j v_j - u_{jx} v_{jx}) dx, \quad \langle Mu, v\rangle = \sum_j \alpha_{1j} d_j \int_0^{l_j} u_j v_j dx,$$

$$\langle N(u), v\rangle = \sum_j d_j \left(\alpha_{2j} \int_0^{l_j} u_j^3 v_j dx + ... + \alpha_{nj} \int_0^{l_j} u_j^{2n-1} v_j dx\right),$$

where $\langle \cdot, \cdot \rangle$ is the inner product on $\mathbf{L_2}(\mathbf{G})$. For all $\alpha_{2j}, \alpha_{3j}, ..., \alpha_{nj} \in \mathbb{R}$ we have $N \in C^\infty(\mathfrak{U}; \mathfrak{F})$. The operators L and M lie in $\mathcal{L}(\mathfrak{U}; \mathfrak{F})$, that is, are linear and continuous. Moreover, L is Fredholm (that is, $\mathrm{ind\,L} = 0$), while M is compact and $(L, 0)$-bounded whenever $\ker L = \{0\}$ or $\ker L \neq \{0\}$ and all coefficients α_{1j} are nonzero and of the same sign.

Let us state the main results as theorems on the solvability of direct and inverse problems in the case of the Cauchy problem and Showalter–Sidorov problem for Hoff's equations on the graph (the first three theorems are proved in [1, 2]).

Theorem 1.

(i) *If* $\ker L = \{0\}$ *then the phase space of (1) is* $\mathfrak{U}$.

(ii) *If* $\ker L \neq \{0\}$*, all* α_{1j} *are nonzero and of the same sign as all nonzero coefficients* α_{sj} *for* $s = 2, ..., n$ *then the phase space of (1) is the simple manifold*

$$\mathfrak{M} = \{u \in \mathfrak{U} : \langle Mu + N(u), \chi_k \rangle = 0, missing\}$$

where $\ker L = \operatorname{span}\{\chi_k : k = 1, 2, ..., l\}$ *is an* $\mathbf{L_2}(\mathbf{G})$*-orthonormal basis of* $\ker L$ *identified with a basis of* $\operatorname{coker} L$.

Denote by $\mathfrak{D}$ the set of admissible values of the vectors φ and ψ for which the solutions to the inverse problem are the coefficients α_j and β_j of the same sign ($\alpha_j > 0$, $\beta_j > 0$ or $\alpha_j < 0$, $\beta_j < 0$ for all j). The structure of the set $\mathfrak{D}$ is described in [2].

Theorem 2.

(i) *If* $\ker L = \{0\}$ *then for all* $u_0 \in \mathfrak{U}$ *and* φ_j, $\psi_j \in \mathbb{R}$ *satisfying* $u_{0j}(0) \neq 0$, $u_{0j}(l_j) \neq 0$, $u_{0j}(0) \neq \pm u_{0j}(l_j)$, $\varphi_j u_{0j}^3(l_j) \neq \psi_j u_{0j}^3(0)$, *and* $\varphi_j u_{0j}(l_j) \neq \psi_j u_{0j}(0)$ *there exists a unique solution to the inverse problem (1)–(4), (6).*

(ii) *If* $\ker L \neq \{0\}$ *then for all* $\varphi, \psi \in \mathfrak{D}$ *and* $u_0 = (u_{01}, u_{02}, ..., u_{0j}, ...) \in \mathfrak{U}$ *satisfying* $u_{0j}(0) \neq 0$, $u_{0j}(l_j) \neq 0$, $u_{0j}(0) \neq \pm u_{0j}(l_j)$, *and*

$$\left\langle \sum_j \frac{d_j}{\mathfrak{d}_j} \int_0^{l_j} \left((\varphi_j u_{0j}^3(l_j) - \psi_j u_{0j}^3(0))u_{0j} + (\psi_j u_{0j}(0) - \varphi_j u_{0j}(l_j))u_{0j}^3\right) dx, \chi_k \right\rangle = 0,$$

there exists a unique solution $u \in \mathfrak{U}$, α_j, $\beta_j \in \mathbb{R}\backslash\{0\}$ *to the inverse problem (1)–(4), (6) with* $\alpha_j\beta_j \in \mathbb{R}_+$.

Theorem 3.

(i) *If* $\ker L = \{0\}$ *then for all* $u_0 \in \mathfrak{U}$ *there exists a unique solution to the Showalter–Sidorov problem (1)–(3), (5).*

(ii) *If* $\ker L \neq \{0\}$ *and all coefficients* $\alpha_{sj} \neq 0$ *for* $s = 1, ..., n$ *are of the same sign, then for all* $u_0 \in \mathfrak{U}$ *there exists a unique solution to the Showalter-Sidorov problem (1)–(3), (5).*

Theorem 4.

(i) *If* $\ker L = \{0\}$ *then for all* $u_0 \in \mathfrak{U}$, φ_j, $\psi_j \in \mathbb{R}$ *satisfying* $u_{0j}(0) \neq 0$, $u_{0j}(l_j) \neq 0$, $u_{0j}(0) \neq \pm u_{0j}(l_j)$, $\varphi_j u_{0j}^3(l_j) \neq \psi_j u_{0j}^3(0)$, *and* $\varphi_j u_{0j}(l_j) \neq \psi_j u_{0j}(0)$ *there exists a unique solution to the inverse problem (1)–(3), (5), (6).*

(ii) *If* $\ker L \neq \{0\}$ *then for all vectors* $\varphi, \psi \in \mathfrak{D}$ *and* $u_0 \in \mathfrak{U}$ *satisfying* $u_{0j}(0) \neq 0$, $u_{0j}(l_j) \neq 0$, *and* $u_{0j}(0) \neq \pm u_{0j}(l_j)$ *there exists a unique solution* $u \in \mathfrak{U}$, α_j, $\beta_j \in \mathbb{R}\backslash\{0\}$ *to the inverse problem (1)–(3), (5), (6) with* $\alpha_j\beta_j \in \mathbb{R}_+$.

Proof. (ii) Suppose that $u_0 \in \mathfrak{U}$ and the conditions $u_{0j}(0) \neq 0$, $u_{0j}(l_j) \neq 0$, $u_{0j}(0) \neq \pm u_{0j}(l_j)$ are fulfilled. Then there exists a unique pair of coefficients α_j and β_j satisfying (6). The condition $\varphi, \psi \in \mathfrak{D}$ ensures that $\alpha_j\beta_j > 0$ for all j. Moreover, by the construction of $\mathfrak{D}$, all coefficients α_j are nonzero and of the same sign as the nonzero coefficients β_j. Hence, the hypotheses of claim (ii) of Theorem 3 hold, and so there exists a unique solution $u \in \mathfrak{U}$, α_j, $\beta_j \in \mathbb{R}\backslash\{0\}$ to the inverse problem (1)–(3), (5), (6) with $\alpha_j\beta_j \in \mathbb{R}_+$.

□

2. The Results of Simulations

Basing on the theoretical results for Hoff's model on a graph, we wrote a software bundle in the symbolic algebra package Maple 15.0. It enables us to:

(1) find numerical solutions to direct and inverse problems with given coefficients λ_j, α_{ij}, φ_j, and ψ_j as Galerkin sums of the first few eigenfunctions;

(2) draw the graphs of the solutions to Hoff's model on a graph;

(3) draw the phase space of Hoff's model on a graph.

Example 1. *Take the graph* $\mathbf{G}$ *with three vertices* V_1, V_2, *and* V_3 *and two adjacent edges* E_1, *of length* $l_1 = \pi$ *and area* $d_1 = 1$ *of the cross-section, and* E_2, *of length* $l_2 = \pi$ *and area* $d_2 = 1$ *of the cross-section.*

Consider on $\mathbf{G}$ the equations of Hoff's model with $\lambda_1 = 1$, $\lambda_2 = 1$, $\alpha_{11} = -10$, $\alpha_{21} = -0,5$, $\alpha_{12} = -0,5$, $\alpha_{22} = -0,19$:

$$\begin{cases} u_{1t} + u_{1xxt} + 10u_1 + 0,5u_1^3 = 0 \\ u_{2t} + u_{2xxt} + 0,5u_2 + 0,19u_2^3 = 0. \end{cases} \tag{7}$$

The continuity conditions are $u_1(\pi, t) = u_2(0, t)$, and the flow balance conditions are $u_{1x}(0, t) = 0$, $u_{1x}(\pi, t) = u_{2x}(0, t)$, and $u_{2x}(\pi, t) = 0$. Seek the solution to (7) as the Galerkin sums

$$u_1^N(x,t) = \sum_{i=1}^{N} v_i(t)\xi_i(x), \quad u_2^N(x,t) = \sum_{i=1}^{N} v_i(t)\zeta_i(x),$$

where $\chi_i(x) = (\xi_i(x), \zeta_i(x))$ are the eigenfunctions of the Sturm–Liouville problem on the graph. For $N = 3$, the eigenvalues and eigenfunctions are

$$\begin{gathered} \mu_1 = 1, \quad \chi_1 = (1, 1), \\ \mu_2 = 0,75, \quad \chi_2 = (\cos(\frac{x}{2}), -\sin(\frac{x}{2})), \\ \mu_3 = 0, \chi_3 = (\cos x, -\cos x), \end{gathered}$$

so we look for the solution in the form

$$\begin{aligned} u_1(x,t) &= v_1(t) + v_2(t)\cos x + v_3(t)\cos(\frac{x}{2}), \\ u_2(x,t) &= v_1(t) - v_2(t)\cos x - v_3(t)\sin(\frac{x}{2}). \end{aligned}$$

In the case $\lambda_1 = \lambda_2 = 1$ claim (ii) of Theorem 1 applies because $0 \in \sigma(L)$.

Multiplying (7) by the functions χ_k for $k = 1, 2, 3$, we obtain the system of differential

Table 1

The numerical solution to (8) with initial data
$v_1(0) = -0,01, v_2(0) = -0,01, v_3(0) = 0,02$

t	$v_1(t)$	$v_2(t)$	$v_3(t)$
0	$-0,01$	-0,00738	0,02
0,05	$-0,010116$	$-0,006685$	0,018122
0,10	$-0,009997$	$-0,006173$	0,016736
0,15	$-0,009748$	$-0,005774$	0,015652
0,20	$-0,00943$	$-0,005443$	0,014757
0,25	$-0,00908$	$-0,005159$	0,013985
0,30	$-0,008718$	$-0,004905$	0,013296

equations

$$\left\{\begin{array}{l} 0,4786v_2^2(t)v_3(t)+0,248v_3^3(t)+6,2v_3(t)+0,62v_1^2(t)v_3(t)+ \\ +1,736v_1(t)v_2(t)v_3(t)+1,626v_2(t)v_3^2(t)+16,808v_2(t)+ \\ +3,252v_1^2(t)v_2(t)+0,813v_2^3(t)+1,626v_1(t)v_3^2(t)=0, \\ 1,24v_1(t)v_2(t)v_3(t)+18,6v_3(t)+0,413v_3^3(t)+0,868v_2^2(t)v_3(t)+ \\ +1,86v_1^2(t)v_3(t)+33,615v_1(t)+3,252v_1(t)v_3^2(t)+6,283\dot{v}_1(t)+ \\ +2,168v_1^3(t)+1,626v_2(t)v_3^2(t)+3,252v_1(t)v_2^2(t)=0, \\ 0,868v_1(t)v_2^2(t)+18,6v_1(t)+1,24v_1(t)v_3^2(t)+0,744v_2(t)v_3^2(t)+0,62v_1^2(t)v_2(t) \\ +0,159v_2^3(t)+6,2v_2(t)+0,62v_1^3(t)+1,626v_2^2(t)v_3(t)+3,252v_1^2(t)v_3(t) \\ +16,808v_3(t)+3,252v_1(t)v_2(t)v_3(t)+0,813v_3^3(t)+2,356\dot{v}_3(t)=0. \end{array}\right. \tag{8}$$

Figure 1.a depicts its phase space.

Let us solve the Showalter–Sidorov problem for (8) with the data $v_1(0) = -0,01$, $v_2(0) = -0,01$, and $v_3(0) = 0,02$, which corresponds to the initial condition

$$u_1(x,0) = -0,01 - 0,01\cos x + 0,02\cos(\frac{x}{2})$$
$$u_2(x,0) = -0,01 - 0,01\cos x - 0,02\sin(\frac{x}{2})$$

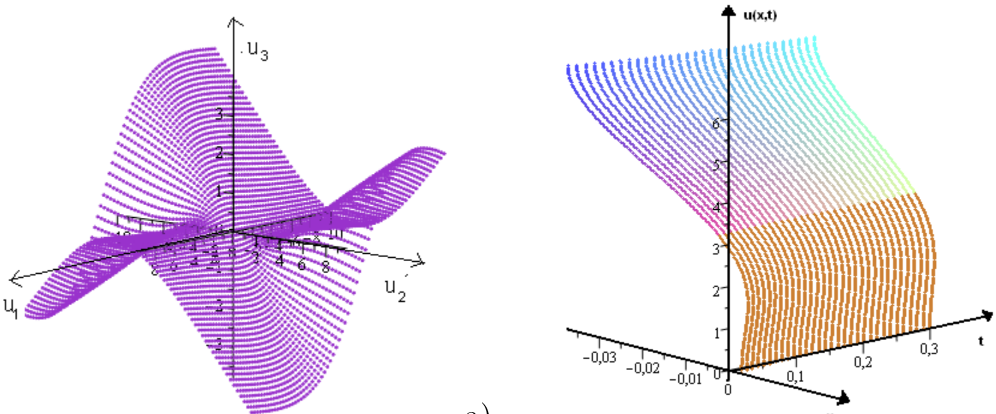
for (7). Table 1 lists some values of the solution to (8) in a neighbourhood of the point $t = 0$, while Figure 1.b depicts its graph.

Example 2. *Take the graph* $\mathbf{G}$ *with three vertices* V_1*,* V_2*, and* V_3 *and two adjacent edges* E_1*, of length* $l_1 = \pi$ *and area* $d_1 = 7$ *of cross-section, and* E_2*, of length* $l_2 = \pi$ *and area* $d_2 = 1$ *of cross-section.*

Consider on $\mathbf{G}$ the equations of Hoff's model with $\alpha_{11} = -1$, $\alpha_{21} = -0,5$, $\alpha_{12} = -0,7$, $\alpha_{22} = -0,9$, $\lambda_1 = 1$, $\lambda_2 = 4$:

$$\left\{\begin{array}{l} u_{1t} + u_{1xxt} + u_1 + 0,5u_1^3 = 0 \\ 4u_{2t} + u_{2xxt} + 0,7u_2 + 0,9u_2^3 = 0. \end{array}\right. \tag{9}$$

The continuity condition is $u_1(\pi,t) = u_2(0,t)$, and the flow balance conditions are $7u_{1x}(0,t) = 0$, $7u_{1x}(\pi,t) = u_{2x}(0,t)$, and $u_{2x}(\pi,t) = 0$. Seek the solution as the Galerkin



a) b)

Fig. 1. (a) The phase space of (1) with $\lambda_1 = \lambda_2 = 1$; (b) The solution to (8) with $v_1(0) = -0,01, v_2(0) = -0,01, v_3(0) = 0,02$

sums

$$u_1^N(x,t) = \sum_{i=1}^{N} v_i(t)\xi_i(x), \quad u_2^N(x,t) = \sum_{i=1}^{N} v_i(t)\zeta_i(x),$$

where $\chi_i(x) = (\xi_i(x), \zeta_i(x))$ are the eigenfunctions of the Sturm–Liouville problem on the graph. For $N = 2$ the eigenvalues and eigenfunctions are

$$\mu_1 = 0, \chi_1 = (\cos x, -\cos 2x),$$
$$\mu_2 = \frac{15}{16}, \chi_2 = \big(\cos\frac{x}{4}, \cos\frac{7x}{4} - \sin\frac{7x}{4}\big),$$

and so we look for the solution in the form

$$u_1(t,x) = v_1(t)\cos x + v_2(t)\cos\frac{x}{4},$$
$$u_2(t,x) = -v_1(t)\cos 2x + v_2(t)(\cos\frac{7x}{4} - \sin\frac{7x}{4}).$$

In the case $\lambda_1 = 1$ and $\lambda_2 = 4$ the hypotheses of claim (ii) of Theorem 1 hold because $0 \in \sigma(L)$.

Multiplying (11) by the functions χ_k for $k = 1, 2$, we obtain the system of differential equations

$$\begin{cases} -7v_1^2(t)v_2(t) + 93v_1(t)v_2^2(t) + 33v_1^3(t) + 963v_1(t) + 30v_2(t) + 4v_2^3(t) = 0, \\ 142v_2(t) + 136\dot{v}_2(t) - 0,2v_1^3(t) + 3v_1(t) + 9v_1^2(t)v_2(t) + 1v_1(t)v_2^2(t) + 5v_2^3(t) = 0. \end{cases} \tag{10}$$

Figure 2.a depicts the phase space of (10).

Let us solve the Showalter–Sidorov problem for (10) with the data $v_1(0) = -0,01$, $v_2(0) = -0,01$, and $v_3(0) = 0,02$, which corresponds to the initial condition

$$u_1(x,0) = -0,01 - 0,01\cos x + 0,02\cos(\frac{x}{2}),$$
$$u_2(x,0) = -0,01 - 0,01\cos x - 0,02\sin(\frac{x}{2})$$

Table 2

The numerical solution to (10) with the initial condition $u_1(0) = -0,032$, $u_2(0) = 1$

t	$u_1(t)$	$u_2(t)$
0	$-0,0326786$	1
0,4	$-0,020976$	0,65291
0,8	$-0,013661$	0,428574
1,0	$-0,011055$	0,347559
1,2	$-0,008956$	0,281959
1,4	$-0,00726$	0,228795
1,6	$-0,005889$	0,185683
1,8	$-0,004778$	0,150710
2,0	$-0,003877$	0,122333

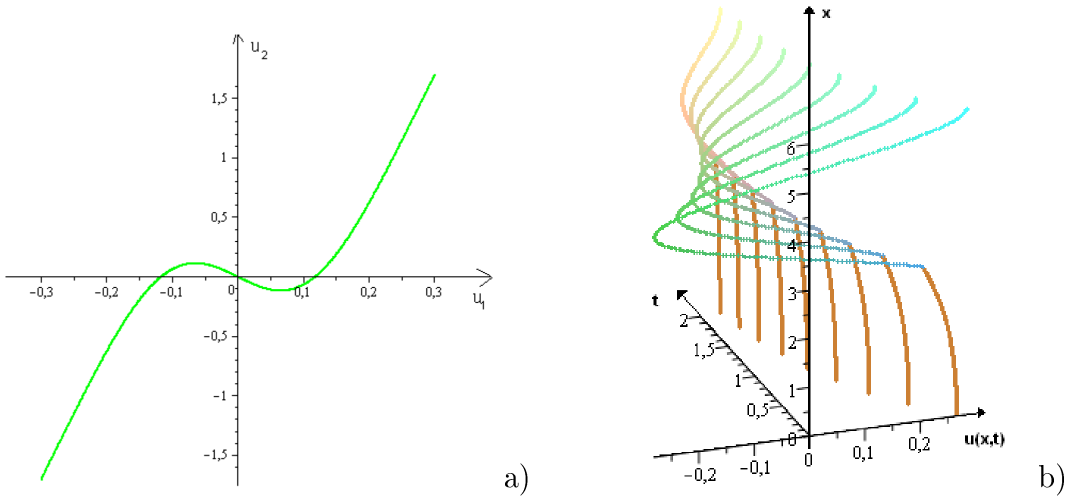


Fig. 2. (a) The phase space of (10) with $\lambda_1 = 1$ and $\lambda_2 = 4$; (b) The solution to (10) with $u_1(0) = -0,032$, $u_2(0) = 1$, and $\lambda_1 = 1$, $\lambda_2 = 4$

for (11). Table 2 lists some values of the solution to (10) in a neighbourhood of the point $t = 0$, while Figure 2.b depicts its graph.

Example 3. *Take the finite oriented graph* **G** *with three vertices V_1, V_2, and V_3, and with adjacent edges E_1, of length $l_1 = \pi$ and area $d_1 = 7$ of cross-section, and E_2, of length $l_2 = \pi$ and $d_2 = 1$ area of cross-section.*

Consider on Hoff's equations **G** with the coefficients $\lambda_1 = 1$ and $\lambda_2 = 4$:

$$\begin{cases} u_{1t} + u_{1xxt} + \alpha_{11}u_1 + \alpha_{21}u_1^3 = 0 \\ 4u_{2t} + u_{2xxt} + \alpha_{12}u_2 + \alpha_{22}u_2^3 = 0. \end{cases} \quad (11)$$

The continuity condition is $u_1(\pi, t) = u_2(0, t)$, and the flow balance conditions are $7u_{1x}(0, t) = 0$, $7u_{1x}(\pi, t) = u_{2x}(0, t)$, and $u_{2x}(\pi, t) = 0$. We have to find the numerical solution to the inverse problem with Showalter–Sidorov initial conditions with $\varphi_1 = -0,06$,

$\varphi_2 = -0,6$, $\psi_1 = -0,096$, $\psi_2 = -0,96$,

$$u_{10}(x) = -0,1\cos x + 0,3\cos\frac{x}{4},$$
$$u_{20}(x) = 0,1\cos 2x + 0,3(\cos\frac{7x}{4} - \sin\frac{7x}{4}).$$

Theorem 2 implies that

$$u_{10}(0) = 0,2, \quad u_{10}(\pi) = 0,05(2+3\sqrt{2}) \approx 0,312,$$
$$u_{20}(0) = 0,4, \quad u_{20}(\pi) = 0,1(1+3\sqrt{2}) \approx 0,524,$$

$\varphi_1 = -0,06$, $\varphi_2 = -0,6$, $\psi_1 = -0,096$, $\psi_2 = -0,96$. Therefore, $\mathfrak{d}_1 = 0,003579 \neq 0$ and $\mathfrak{d}_2 = 0,024 \neq 0$, so the hypotheses of claim (ii) of Theorem 2 hold. The unknown coefficients $\alpha_1 = -0,295$, $\alpha_2 = -1,03$, $\beta_1 = -0,134$, and $\beta_2 = -2,9$ are nonzero and of the same sign.

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МОДЕЛЬ ХОФФА НА ГЕОМЕТРИЧЕСКОМ ГРАФЕ. ВЫЧИСЛИТЕЛЬНЫЙ ЭКСПЕРИМЕНТ

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Целью статьи является численное исследование задачи Шоуолтера–Сидорова (Коши) и обратной задачи для обобщенной модели Хоффа. На основе метода фазового пространства и модифицированного метода Галеркина разработаны алгоритмы численного решения начально-краевой и обратной задач для указанной модели и реализована в виде комплекса программ в системе компьютерной математики Maple 15.0. Уравнение Хоффа, заданное на каждом ребре графа, описывает выпучивание двутавровой балки, а модель Хоффа описывает динамику конструкции из двутавровых балок. Обратная задача состоит в определении неизвестных коэффициентов по результатам дополнительных измерений, показывающих изменение скорости динамики выпучивания в начале и конце балки в начальный промежуток времени. Проведенное исследование основано на результатах теории полулинейных уравнений соболевского типа, поскольку начально-краевая задача для соответствующей системы дифференциальных уравнений в частных производных сводится к абстрактной задаче Шоуолтера – Сидорова (Коши) для уравнений соболевского типа. В приведенных примерах вычисляются собственные значения и собственные функции для оператора Штурма–Лиувилля на графе, находится решение в виде галеркинской суммы по нескольким первым собственным функциям.

Ключевые слова: уравнение соболевского типа; модель Хоффа.

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